Multipath Fading Effects on Digital Microwave Links and Countermeasures

by

Habib Ben Said Gharbi

A Thesis Presented to the

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To My Dear Brothers Samir And Kamel,

And To My Best Friend Lotfi

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ملخص

ظاهرة تعدد المسارات في اتصالات الموجات الدقيقة الرقمية وطرق معالجتها

تعاني أنظمة اتصالات الموجات الدقيقة من تعدد المسارات الناتج عن الأحوال الجوية غير العادية .
وتنقسم ظاهرة تعدد المسارات الى نوعين : التضاؤل الموجي العادي والذي يسبب تدهورا في فاعلية نظام الاتصالات أكثر من تأثير التشويش العادي ، والتضاؤل الموجي المعتمد على الذبذبة الذي يشكل عاملا أساسيا في تردي نظام الاتصالات من خلال تداخل الرموز الرقمية .
ان المقاومة الفعالة لظاهرة تعدد المسارات تكمن في استعمال نظام المعادل في جهة الاستقبال ، الا أن تعدد تمثيل هذه الظاهرة الجوية يؤدي الى تنوع فعالية أنظمة المعادل المستخدمة .
لقد تم دراسة مشكلة تعدد المسارات مع مختلف صيغ التعديل الرقمية ، وطبقت نتائج ذلك في دراسة تأثير أنظمة المعادل الرقمية في فعالية نظام الاتصالات الواقع تحت تأثير تعدد المسارات .

ABSTRACT

Digital Microwave Systems suffer mainly from multipath fading, which is due to abnormal atmospheric conditions.

There are two types of Multipath Fading, the Flat-Fading which degrades the system performance more than the usually considered Guassian Noise, and the Frequency-Selective Fading which deteriorates drastically the signal through Intersymbol Interference generation.

The effective tool to combatt MPF lies in the implementation of Equalizer systems at the receiving end, however, as the existing MPF channel modelling functions are numerous, the treatement of the Equalizers with these channel models exhibits different performances.

The problem of multipath fading has been studied for different digital modulation schemes. The results were used to study the effects of digital equalization methods on the system performance in the presence of multipath phenomena.

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NOTATIONS

ASK : Amplitude Shift Keying

AWGN : Additive White Gaussian Noise

BB : Baseband

BER : Bit Error Rate

BW : Bandwidth

Dm : Mean square Distortion

Dp : Peak Distortion

FF : Flat Fading

FSF : Frequency Selective Fading

FSK : Frequency Shift Keying

ISI : Intersymbol Interference

M-QAM: M-ary Quadrature Amplitude Modulation

MPF : Multipath Fading

M-PSK: M-ary Phase Shift Keying

MSE : Mean Square Error

Pe : Probability of error

PSK : Phase Shift Keying

S/N : Signal to Noise ratio

Z.F : Zero Forcing

CHAPTER ONE

INTRODUCTION

1.1: Historic Overview

During the last twenty years, Digital communication systems have acquired wide applications, while Analog systems became more restricted to certain areas due to economical factors. This rapid and tremendous change is due to the rapid progress in digital technology and particularly in digital computers which requires the installation of suitable and compatible communication and transmission systems to handle the data transfer.

Digital radio is becoming the most attractive terrestrial link to accomodate this new communication era. Economically speaking, short haul digital radio is less expensive than the analog 'FM' one, however, Digital microwave systems are suffering some problems which degrade their overall performance. Problems encountered in such sytems include thermal noise generated by the resistive parts of the electronic equipments in both the transmitting and receiving ends, cochannel interference due to channel bandwidth limitation and other impairments like the non-linearities of the RF power amplifiers. These degradation factors have been mostly overcome by a proper design of microwave links and by increasing the fade margin to countereffect the signal level decrease at the receiver's input.

The terrestrial microwave communication system uses the atmosphere as the transmission medium,and the information flows to the receiving end in a line of sight. The atmosphere is naturally characterized by a random climatic conditions, affected by temperature ,pressure, humidity, and existing particles and gasious distributions. These typical factors generate a refractivity index profile, which is linear in normal conditions, and posseses some negative sharpness at a certain altitude in abnormal conditions,resulting in a multipath propagation state.

This anomalous propagation occurs typically in a calm summer evening when normal atmosphere turbulence is minimal, thus permitting tropospheric layering with different refractivity indices, hence the signal, once transmitted, is faced by different media generating signal paths multiplicity with different relative amplitudes and delays. The received signal is therefore composed of many rays with different characteristics causing detection process disruption. During the day, rising wind mixes the atmosphere and reestablish the smooth linear index profile.

A vast amount of research has been carried out to understand and to model the multipath phenomenon, leading to some rigorous empirical equations showing the statistical distributions of the model different parameters , ie signal level attenuation and delays associated with different rays. This probabilistic models have enabled system designers to estimate the microwave link performance by system outage, ie unavailability evaluation.

Multipath fading 'MPF', usually occurs into two manners:

- Flat-fading 'FF', or signal level depression, depicted mostly in analog microwave links.

- Frequency- selective fading 'FSF', which is considered the main source for the severe corruption of digital microwave communication sysems. This type of fading manifests through signal amplitude and delay distortions, giving rise to intersymbol interference.

FSF is the subject of current research in Digital Radio because the FF parameter is far away to give a correct estimation on the system performance.

The flat-fading impact has been well minimized through two main protection systems:

- An increase of signal power at the transmitting end to widen the flat-fading margin.

- The implementation of diversity techniques . This idea was favou- red by the fact that the main effective rays are statistically independent.

The diversity systems encompasses:

-Space diversity

-Frequency diversity

-Time diversity

-Polarisation diversity

Although Space and Frequency diversity techniques are widely used, frequency diversity becomes more restricted due to the limited Microwave spectrum.

1.2: Littereture Review and Problem Formulation

Frequency-Selective Fading has been the subject of extensive simulation research and field work to understand its behaviour and impacts on bit error rate'BER',[1-4]. FSF manifests through in-band distortion generation, consisting of two components, the amplitude dispersion and delay distortion. The experimental work has demonstrated the two MPF components dependence on frequency As an example, an 8 dB dispersion during a 27 dB fade has caused a loss of synchronisation in the microwave system for more than 20 s with a BER > 0.1, and a time delay distortion of 0.6 ns/MHz approximately, [4].

MPF which originates from the multiplicity of rays at the receiving end, is modelled by a channel response with a constructive and destructive vector addition of the received rays. This is illustrated by Fig(1.1).

The work in this thesis concentrates on deriving the probability of error expressions during severe fading conditions, and on the two ray fading channel model.The performance of the optimum equalizers are then analysed with the multipath phenomena. Both theoritical and numerical approaches will be applied in this work.

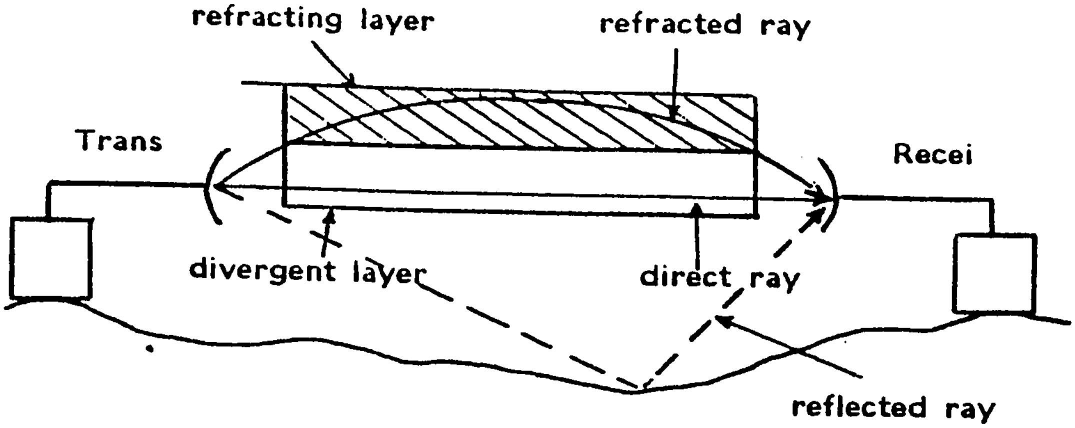
In chapter 1, the litereture is surveyed and detailed outlines of the work is given.

Chapter 2 is devoted to the treatement of the probabilistic models. These models have been extensively treated and tested by field data, some of them have found wide acceptance, like the three-ray model of Rummler [3], and the Polynomial model of Greenstein [8]. This Chapter makes an overview on these models and the statistical distributions of the different parameters involved.

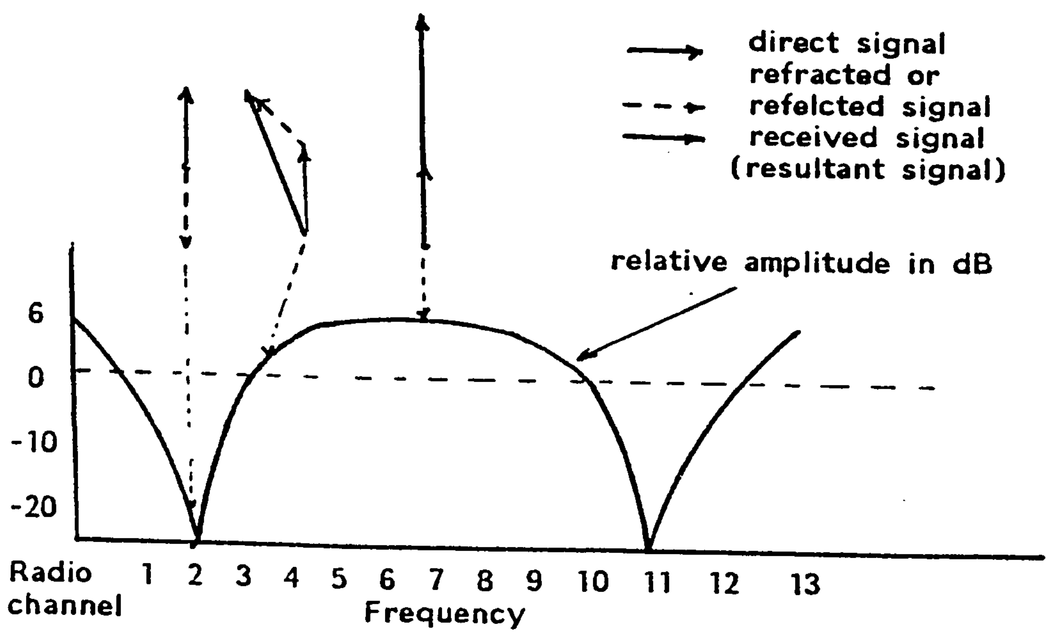
The Flat-Fading component is considered as a white noise, and a fading margin has been estimated to overcome its effects, [5,7,10,12]. Severe FF can occur due to extreme climatic conditions and degrades the system performance beyond what expected.A thorough study is to be made for some efficient binary and M-ary modulation schemes, to evaluate the associated probability of errors, which is the subject of chapter 3. Also the detrimental effects of FSF on worsening the system performance are treated in chapter 3.

In chapter 4, we apply the conventionnal countermeasures to investigate their abilities in combatting MPF effects. they consist of some equalizers at the receiving end. The new trend in digital microwave communications is to implement appropriate optimum equalizers [13], the Z.F equalizer which minimize the peak distortion of the data stream, and the minimum mean square error equalizer which minimizes the MSE or ISI. the tap cofficients are to be estimated . the same treatment is also applied to MPF channels.

The investigations presented in this thesis are concluded in chapter 5, by a study of a hypothetical microwave link, which includes the appropriate systems to countereffect MPF phenomena.



(a)Refraction or reflection during multipath fading



(b) Amplitude characteristics

Fig(1.1): Multipath Fading channel

CHAPTER TWO

MULTIPATH FADING CHANNEL MODELS

2.1: Multipath Fading Overview

In order to evaluate the performance of digital radios, it is necessary to model the impacts of the atmospheric anomalies and abnormal conditions on digital transmission. The model would represent the effects of the propagation defects which may occur on the transmission path.

The study of propagation effects on LOS links began with the introduction FM systems, much work has been done to understand the behaviour and the characteristics of the channel and to model it in a mathematical or empirical form in order to evaluate the system performance through outage estimation. However, the introduction of digital radios activated this work because digital radios were found to be more sensitive to multipath propagation than FM systems.

Unlike FM systems, in which multipath fading phenomena manifests through constant level depression over signal bandwidth, called by Flat Fading, digital radio is accompanied by frequency selective fading which causes deeper amplitude depression over some frequency bands than others, thus affecting seriously the received signal detection.

High capacity digital radios operate over network of paths in assigned frequency bands ranging from 2 up to 15 GHz. These bands are subdivided into channels with bandwidths of about 0.5 %, thus 20 MHz, 40 MHz and 60 MHz are assigned to 4,8 and 12 GHz respectively. Typically, Radio links in the bands below 10 GHz use high directivity antennae, with nearly 1 degree beamwidth, and with tower heights of 50 to 100 meters. The path length of the link is about 50 Km .

During Multipath event, the atmosphere is layered, and energy radiated into space is received through different rays. The receiver sees a weighted sum of time shifted replicas of the transmitted signals, the impulse response of such channel can be modeled by the following expression :

$$h(t) = \sum_{k=0}^{N} \alpha_k \, \delta(t - \tau_k) \qquad (2.1)$$

The corresponding frequency response, ie the channel voltage transfer function is given by :

$$H(w) = \sum_{k=0}^{N} \alpha_k \, e^{-jw\tau_k} \qquad (2.2)$$

The channel model should fit to a high degree of accuracy the characteristics and instantaneous variations of the channel in the appropriate frequency interval.

There are two ways of modeling the channel, the first describes the physical propagation, and called the atmospheric model, it is usually derived from optical theory work, as the ray tracing method and employed when MPF is treated from electromagnetism approach. The second, called the channel model, represents in fact the frequency response of the channel, and is used to evaluate the performance of LOS links from communication systems approach.

The channel transfer function can be written in a magnitude -phase form

$$H(w) = | H(w) | e^{j\phi(w)} \qquad (2.3)$$

The voltage attenuation in dB is given by :

$$A(w) = -20 \log |H(w)| \qquad (2.4)$$

and the delay distortion or group delay by

$$D(w) = - \frac{\delta\phi}{\delta w} \qquad (2.5)$$

In practical way, the model should be associated with three basic components in order to provide the means to estimate the fraction of time in which the system is not achieving its reliabitity, these components are :

- A channel modeling function which approximates H(w) over the frequency interval of interest by suitable choices of the parameters of the function.
- The joint probability distribution for these parameters, conditionned on the presence of MPF.
- The scale factor which accounts for the observation period, when multiplied by the joint probability of the parameters, it gives the estimated system outage during the worst fading month or per year. The scale factor should be derived from the data base gathered during the experimental work.
- The occurrence factor, which takes into consideration the topography of the terrain,the climatic conditions and the atmospheric behaviour,there exists a relation between the scale factor and occurrence factor.

2.2: The Channel Models

The tremendous research to model MPF event has resulted in various models, which depend greatly on the type of radio system employed. The model is different for a link using a diversity system, or adopting dual-polarization scheme, but we concentrate here on non-diversity single -polarization models.

2.3: *The polynomial model*

2.3.1: *Introduction*

One way used to model the channel under MPF effects, is to fit the measured amplitude-frequency responses from a certain operating LOS link, with an appropriate mathematical expression in frequency domain. Although data fitting is at present possible through many mathematical distributions, like exponential and polynomial forms, the latter shows more importance since a high degree of accuracy can be reached by addition of terms until achieving the exact distribution. However, the disadvange of the polynomial models is that it excludes the figure of ray multiplicity from the expression.

As done in [8], let express MPF frequency response as a complex polynomial expanded about the channel center frequency and normalized by its unfaded gain, thus we get :

$$H(w) = A_o + \sum_{n=0}^{N} (A_n + jB_n)(jw)^n \tag{2.6}$$

where the coefficients A_n and B_n 's vary slowly relative to the speed of typical digital radio systems.

at $w=0$, $H(w) = A_o$

where A_o is a real number denoting the median depression or the Flat-Fading level.

Three factors make the polynomial modelling attractive for MPF anomalies:

1- It leads to simple methods in digital signal processing since the term $(jw)^n$ *corresponds the* n^{th} *time* derivative.

2- It leads to a simple adaptive equalization form given by the rational function 1/H(w) which may be easy to realize when the complex zeros of H(w) have negative real parts.

3- The statistical and data fitting approach has led to the conclusion that a first order presentation , N = 1 may be sufficiently accurate for LOS links with carrier frequencies below 15 GHz and hence the channel response function could be characterized by the joint pdf of the coefficients A_o, A_1 and B_1 only.

2.3.2: Model description and statistics

The complete method in extracting the polynomial from the data base and the error analysis are reported in]8,11[, we present here a brief overview on this method.

The frequency response records consist of the quantized values of $-10 \log_{10} |H(w)|^2$ at 23 different frequencies, then the decibel quantity P_i , the data record at i frequency to a power ratio $p_i = 10^{\frac{-P_i}{10}}$, and fitting the sequences of p_i vs frequency with an Mth-order polynomial :

$$q(w) = D_o + wD_1 + \ldots\ldots\ldots + w^M D_M$$

The least-square optimization form has been used to evaluate the coefficient family (D_o ,...... D_M) It was found that for highly selective fadings, the most suitable polynomial order is M = 4, however, for most fading periods, polynomial of order M=2 provides accurate representation.

Let write H(w) in a power gain function :

$$|H(w)|^2 = D'_o + wD'_1 + \ldots\ldots\ldots + w^{2N} D'_{2N}$$

Where (D'_o ,........., D'_{2N}) are simply related to A_n 's and

B_n 's, as an example,

$$D'_o = A_o$$

$$D'_1 = -B_1 + jA_1$$

$$D'_2 = -A_2 - jB_2$$

$$D'_3 = B_3 - jA_3$$

The final step is to match the family $(D'_o , \ldots\ldots\ldots, D'_{2N})$ to $(D_o , \ldots\ldots\ldots, D_M)$ and by choosing the order N of the polynomial H(w), the coefficients A_n*'s* and B_n*'s* are then evaluated.

The model structure, consisting of the transfer function and the parameters pdf's, are as following:

i) The complex transfer function of the channel, normalized by its unfaded gain is

$$H(w) = \begin{cases} 1 & \text{during non-fading periods} \\ A_o - wB_1 + jwA_1 & \text{during } T_M \text{ seconds per heavy fading months} \end{cases} \tag{2.7}$$

ii) By assuming T_M to be proportional to MPF occurrence factor,

$$T_M = [0.11]\ c\ F\ d^3 \tag{2.8}$$

where

[0.11]: a data derived scale factor that vary with path lengths,antennae location, year, etc.

c : the terrain factor ranging from 0.25 to 1.0

F : the carrier frequency in GHz

d : the path length in miles

iii) the joint pdf of A_o, A_1 and B_1 can be represented by:

$$p(a_o,A_1,B_1) = p_A(A_1/a_o)p_B(B_1/a_o)p_a(a_o) \tag{2.9}$$

where

$$a_o = \frac{[20\ \log_{10}A_o - (-21.39)]}{6.562}$$

where

A_o is dimensionless and A_1, B_1 are in units of seconds, furthermore, as can be noticed, A_1 and B_1 are statistically independent on each other, but they depend on the FF component A_o.

iv) The pdf of a_o is nearly gaussian given by

$$p_a(a_o) = \frac{1}{\sqrt{2\pi}} e^{\frac{-1}{2} w^2(a_o)} \frac{\delta w(a_o)}{\delta a_o} \tag{2.10}$$

where $w(a_o)$ is a small non linear term given by

$$w(a_o) = a_o + (0.0742)a_o^2 + (0.0125)a_o^3$$

If $w(a_o) = a_o$, the resulting pdf of a_o would be precisely gaussian with zero mean and unity variance. This is illustrated in Fig(2.1)

v) The conditionnal pdf's of A_1 and B_1 are given by

$$p_A(A_1/a_o) = \frac{1}{2\pi\sigma_A(a_o)} e^{[\frac{-1}{2} (\frac{A_1}{\sigma_A(a_o)})^2]} \tag{2.11}$$

and

$$p_B(B_1/a_o) = \frac{1}{2\pi\sigma_B(a_o)} e^{[\frac{-1}{2} (\frac{B_1}{\sigma_B(a_o)})^2]} \tag{2.12}$$

where

$$\sigma_A(a_o) = Max\ [(0.14),\ (0.309 + 0.13a_o)]\ ns$$

and

$$\sigma_B(a_o) = Min\ [(0.24),\ Max(0.120,\ 0.18 + 0.046a_o)]\ ns$$

The parameters σ_A and σ_B are shown in Fig(2.2).

Although the polynomial has suited other experimental data in other sites, it needs further measurements to reinforce and improve it, because, over wider bandwidths or more selective fading channels ,the first order polynomial would be inedequate and at least a quadratic term in (jw) in H(w) would be needed , this would raise the number of parameters to 5, ie (A_o, A_1, A_2, B_1, B_2) and complicate the statistical modeling process.

2.4: The three-ray model

2.4.1: Introduction

The tremendous investigations led to the proposal of the three-ray model which physically exists ,because usually, many rays are detected at the receiving end, and it can fit well MPF.

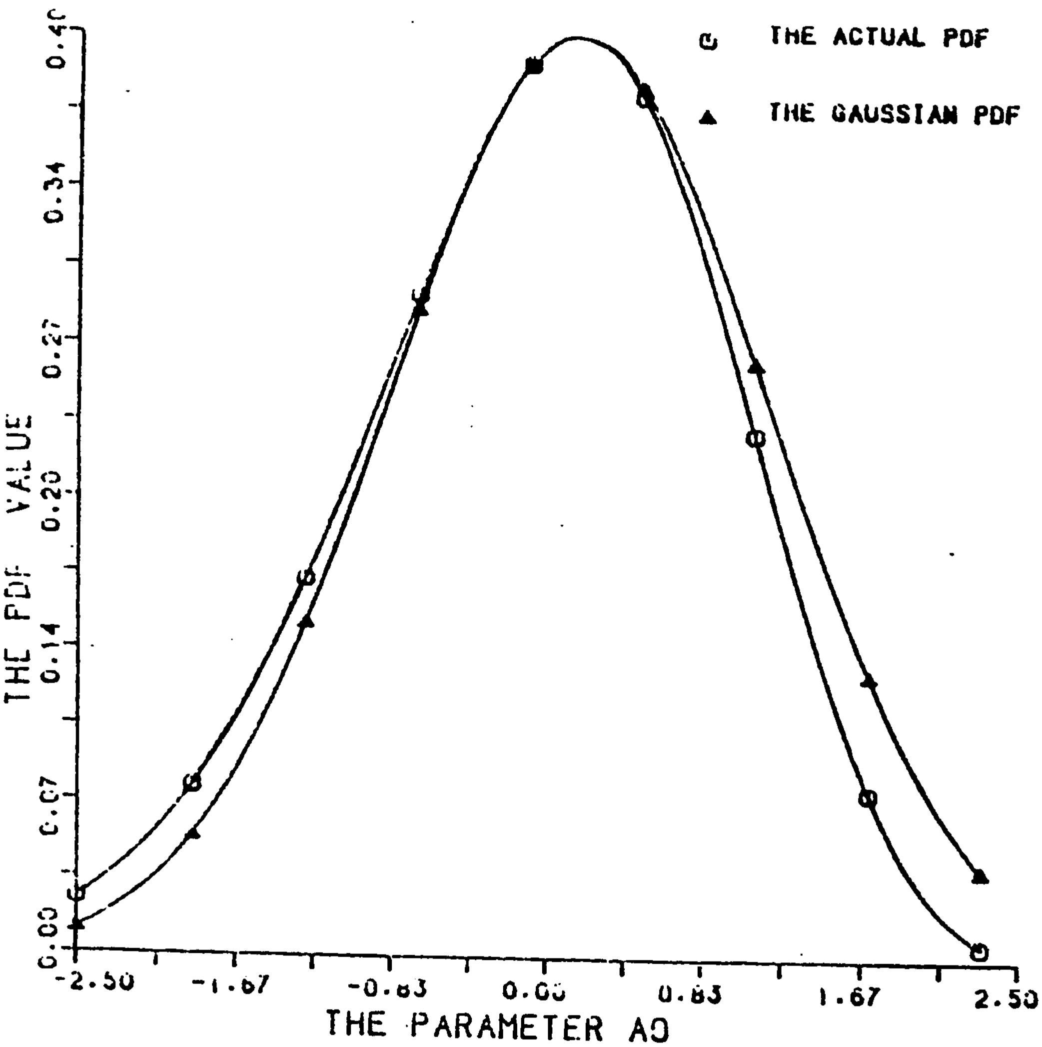


Fig 2.1 Pdf of the parameter ao in the Polynomial Model

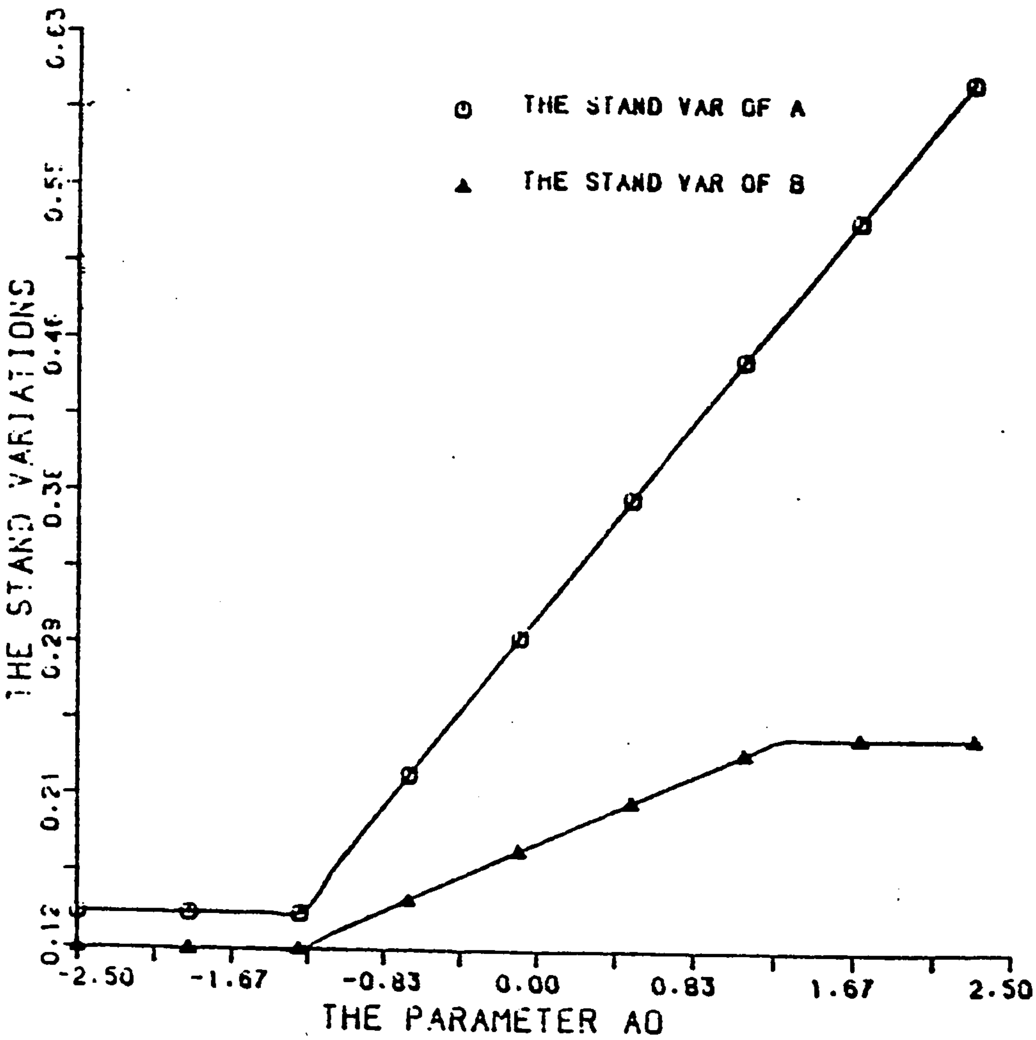


Fig 2.2 Standard Deviations of A1 and B1 variation with ao

The three-ray model has been proposed by Rummler in [3,4] through a statistical approach, and developped from measurements on an unprotected 26.4 mile hop in 6 GHz band in 1977 using 8 PSK modulation scheme.

The voltage transfer function has the following form :

$$H(w) = a[1.0 - b\, e^{-j(w - w_o)\tau}] \qquad (2.13)$$

where the real positive a and b represent the scale and shape of the fade respectively.

τ: the delay difference in the channel

w_o: the radian frequency of the fade minimum

The power transfer function is given by

$$|H(w)|^2 = a^2[1 + b^2 - 2b\cos(w - w_o)\tau] \qquad (2.14)$$

the delay distortion or group delay is expressed by

$$D(w) = -\frac{\delta\phi(w)}{\delta w}$$

where $\phi(w)$ represents the phase H(w). After some mathematical computation, we get

$$D(w) = \frac{b\tau\,[\cos(w - w_o)\tau - b]}{1 + b^2 - 2b\cos(w - w_o)\tau} \qquad (2.15)$$

The channel modelling function H(w) has been found to provide a good fit to almost all measured responses of narrowband radio channels. However, the set of parameters a, b, τ and f_o can not be uniquely determined from a given channel response measurements. To avoid this difficulty, Rummler[3] has reached good channel representation when fixing the delay parameter τ to a certain value, which insures that the period of H(w) in frequency domain is large sufficient compared with the measurements BW, the value of τ was chosen to be 1/6BW, the observation BW was 26.4 MHz, so $\tau \approx 6.31 ns$.

Other works confirmed that the fixed delay model provides a sufficiently accurate representation for narrowband channels such as the 30 MHz BW ones, but some others have followed the factor-of-six rule, that is $\tau = 1/6BW$.

The joint statistics of the model parameters would depend on the choice of τ, but the distribution of the notch frequency is independent of the other parameters. The voltage, power transfer functions are illustrated with $\tau = 6.31$ ns in Figs.(2.3-4)

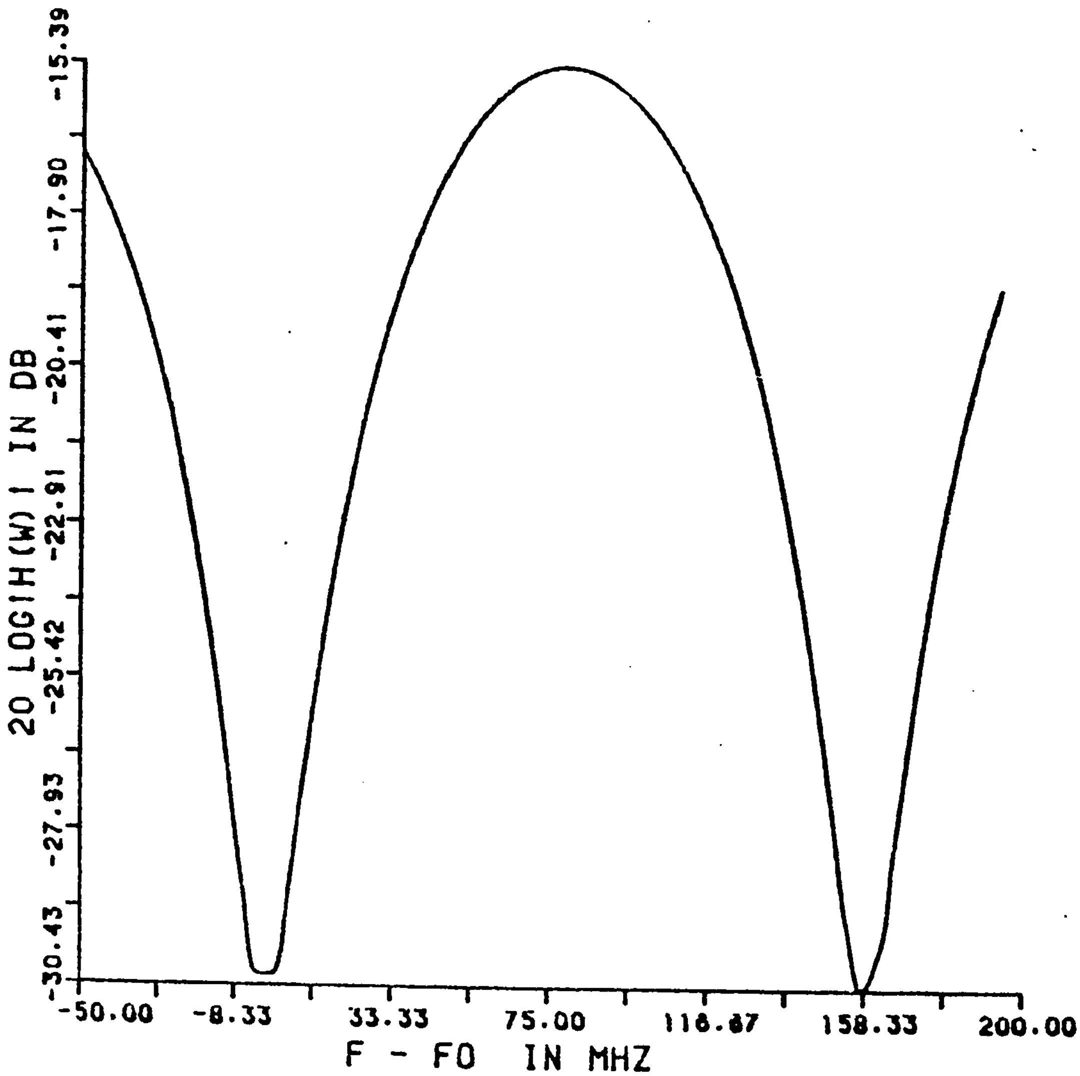


Fig 2.3 Power variation with fo-f (in MHz) for tau= 6.31ns in the Three-Ray model

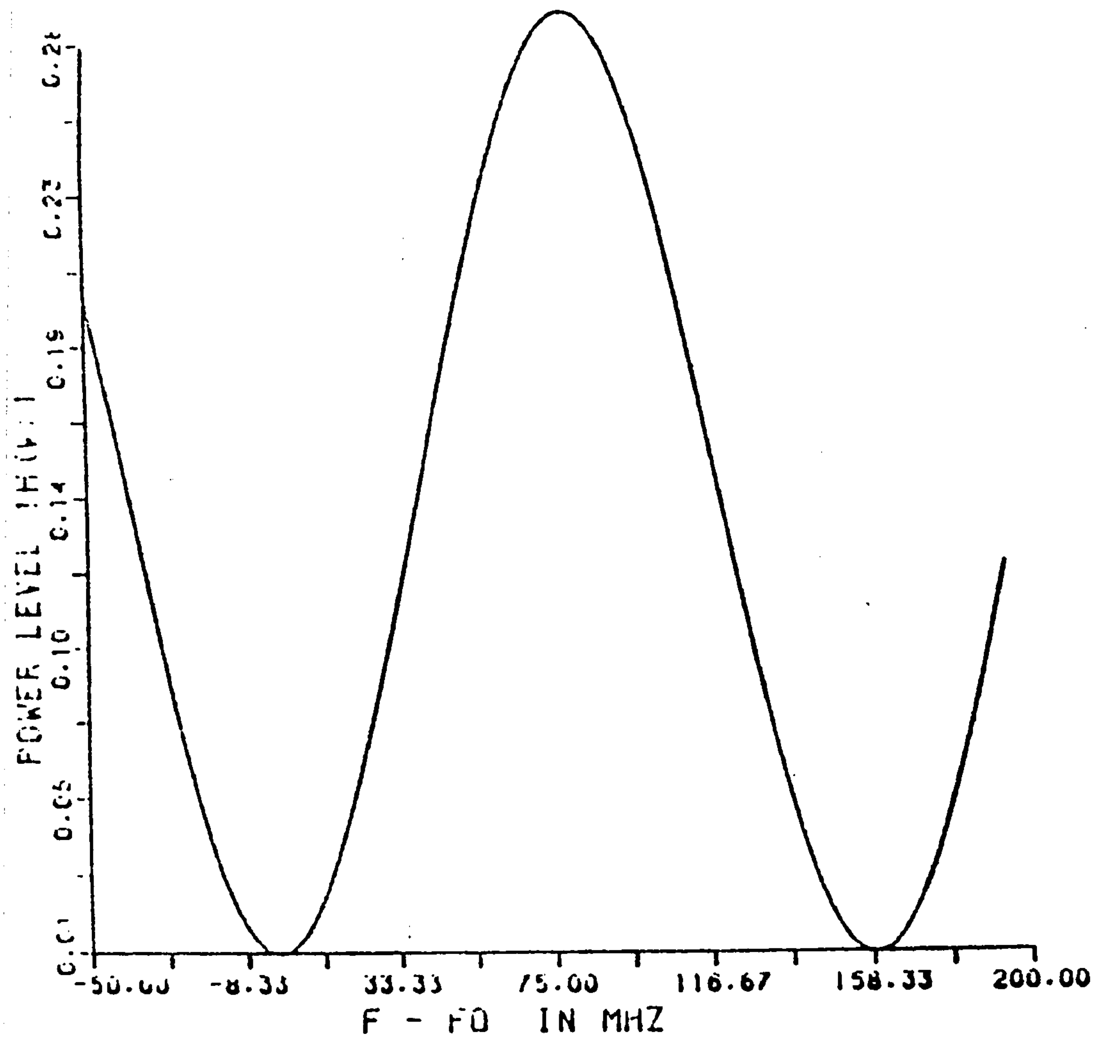


Fig 2.4 Voltage variation with fo-f (in MHz) for tau= 6.31ns in the Three-Ray Model

2.4.2: Model description and statistics

Let the amplitudes of the first, second and third rays and their delays in Eq(2.2) be

$$\alpha_1 = 1 \quad , \quad \tau_1 = 0$$

$$\alpha_2 = a_1 \quad , \quad \tau_2 = \tau_1$$

$$\alpha_3 = a_2 \quad , \quad \tau_3 = \tau_2$$

such that $\tau_2 > \tau_1$ hence

$$|H(w)| = 1 + a_1 e^{-jw\tau_1} + a_2 e^{-jw\tau_2} \tag{2.16}$$

We define the three-ray model by the delay between the two first paths to be sufficiently small, ie

$$(w_2 - w_1)\tau_1 \ll 1$$

where w_2 and w_1 are the highest and the lowest radian frequencies in the band.

so $w_2\tau_1 \simeq w_1\tau_1$

By designating the amplitude of the vector sum of the first two paths by a and the angle by $\phi = w_o\tau-\pi$, we can get the phasor diagram (1).

Let $\tau = \tau_2$ and $a_2 = ab$, the phasor diagram (2) is gotten. The angles x and y can be evaluated

$$x = w_2\tau - (\phi + \pi - w_1\tau) - w_1\tau$$

$$= (w_2 - w_o)\tau$$

and

$$y = \pi - \phi + w_1\tau$$

$$= (w_1 - w_o)\tau$$

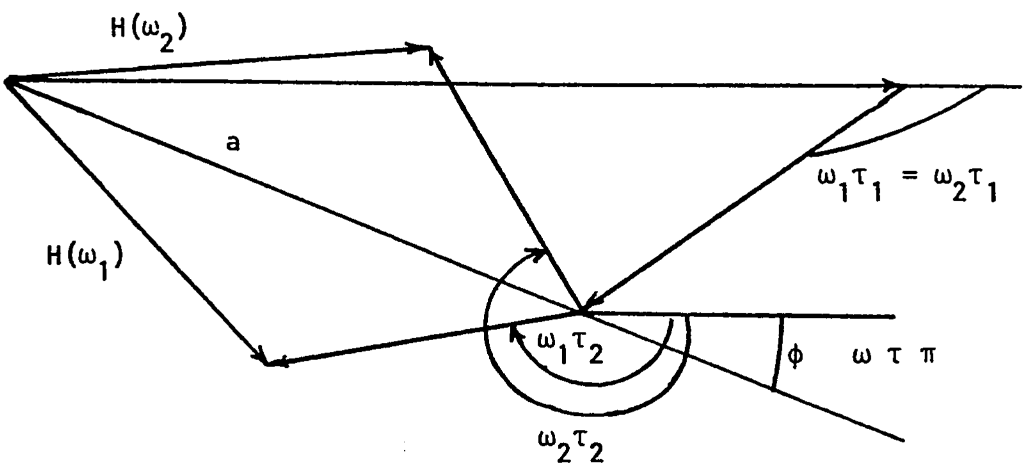
hence, we can write from the phasor diagram (2)

$$H(w_1) = a[1 - be^{-j(w_1 - w_o)\tau}]$$

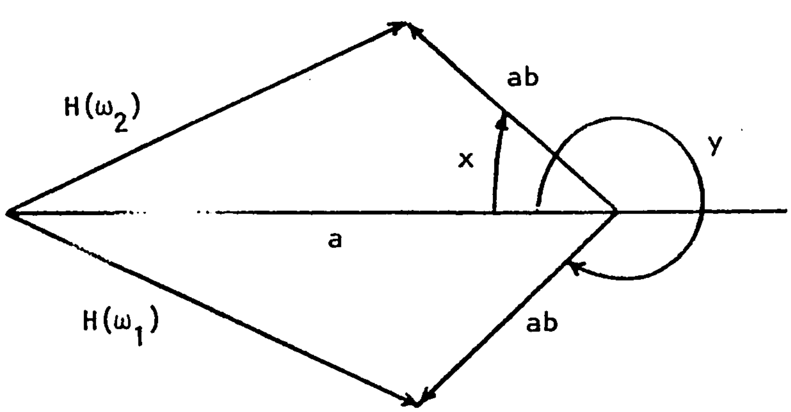
$$H(w_2) = a[1 - be^{-j(w_2 - w_o)\tau}]$$

or generally

$$H(w) = a[1 - be^{-j(w - w_o)\tau}] \tag{2.17}$$



Phasor diagram 1



Phasor diagram 2

The pdf's of the parameters reported in [19], can be used to determine the probability of finding a,b,and f_o in a region in which the prescribed threshold " Ex, BER = 10^{-3} ", is exeeded. Then this probability is multiplied by T_M in Eq(2.8) to estimate the expected number of seconds during the worst fading month or per year.

Notch depth

The parameter b is best described in terms of the number of seconds the ralative notch depth B = -20 log(1-b) exeeds a value x, this is approximated by

$$P(\ B > X\ ) = e^{\frac{-X}{3.8}}$$

and the pdf is given by

$$p_B(X) = \frac{e^{\frac{-X}{3.8}}}{3.8} \tag{2.18}$$

Scale parameter

The distribution of A = -20 log a, has been found to be bependent on B and approximated by

$$P(A>Y/B) = 1 - P_g\left[\frac{Y - A_o(B)}{5}\right]$$

Where P_g is the cumulative distribution of a Gauussian R.V with zero mean and unit variance and A_o (B) is the conditionnal mean of A, so the pdf of A is given by

$$p_A(Y/B) = \frac{1}{\sqrt{2\pi}} e^{[\frac{-1}{2} (\frac{Y - A_o(B)}{5})^2]} \qquad (2.19)$$

Notch frquency

The distribution of f_o has been found to be independent of A and B, let $\pi = 360 f_o \tau$, the relative phase at midband of the second path in the model, the pdf of ϕ per degree is given by

$$p_\phi(\phi) = \left[\begin{array}{ll} \frac{1}{216} & |\phi| \leq 90^0 \\ \\ \frac{1}{1080} & 90^0 < |\phi| \leq 180^0 \end{array} \right. \qquad (2.20)$$

2.5: *The two-ray model*

The two-ray model can be derived from Eq(2.2) by putting N=2

$$H(w) = \alpha_1 e^{-jw\tau_1} + \alpha_2 e^{-jw\tau_2} \tag{2.21}$$

Let

$$\alpha_1 = 1 \quad , \quad \tau_1 = 0$$

$$\alpha_2 = b \quad , \quad \tau_2 = \tau$$

we get

$$H(w) = 1 + b\ e^{-jw\tau} \tag{2.22}$$

The first term represents the main ray and the second is the dominant interfering ray with a relative amplitude and delay b and τ . repective ly, the frequency w is measured at RF. This model has been adopted in th earliest work, but later on, a random phase &cph. component has been added to the delayed ray, this is achieved through the introduction of a notch frequency offset, so the last function is transferred to :

$$H(w) = 1 + b\ e^{-jw\tau\ -\ \phi}$$

with $\phi = w_o\tau - \pi$

we get

$$H(w) = 1 - b\, e^{-j(w - w_o)\tau} \tag{2.23}$$

H(w) depends, as seen from the last equation, only on the frequency difference, which allows w to be measured from any convenient frequency, either RF or IF center frequency.

The model form describes in reality the depression event without refere- nce. That is, it does not show explicitely the level from which the depression varies, this has suggested to some authors to introduce a constant factor a to the modelling function to represent the median depression or the FF component. Hence

$$H(w) = a[1 - be^{-j(w - w_o)\tau}] \tag{2.24}$$

This form is similar as can be noticed, to the three-ray model form, but the parameters meanings are different.

It has been considered in the two-ray model that the parameters are statistically independent of each other, this result was derived from simple approximation to the atmospheric model of propagation.

The pdf of β and τ are given by [16]

$$p_{\beta}(\beta) : \text{uniformly distributed in } [0,1] \tag{2.25}$$

$$p_{\tau}(\tau) = (\tau/\tau_o) \exp(-\tau/\tau_o)\, u(\tau) \tag{2.26}$$

where $\tau_o = E(\tau)$

The next work will be based upon the two-ray model for simplicity.

CHAPTER THREE

MULTIPATH-FADING IMPACT ON DIGITAL MODULATIONS

3.1: The Gaussian Noise Effect On Digital Modulations

The Microwave communication system is obviously a band-pass channel which requires the use of an efficient digital modulation technique. The choice of a certain modulation scheme is usually dictated by many facto- rs such as channel bandwidh availability, transmission rate, the allowed probability of symbol error, power requirements and the complexity of transmission equipments.

Generally, the communication system is optimized to maximize the S/N ratio at the input of the receiver,and hence minimizing the probability of error. For a transmitted signal affected mainly by white noise, the optimum filter is the practical correlator filter.

We present first the probability of bit error expression evaluated for some binary modulation schemes in presence of gaussian noise,[9], This will give us a comparison basis to decide on the optimum modulation scheme. However, required

BW, transmission power and comp- lexity of equipment, form together, with the probability of error, the set of criteria for the decision-makers.

The Error Probability Expressions for Coherent Modulations are reported in Appendix I.

The Pe variation for coherent and non-coherent Binary Modulations are shown in Fig(3.1), the superiority of PSK over the other coherent and non-coherent modulation schemes is clear.

3.2: *Flat-Fading Effect On Binary Modulations*

3.2.1: *PSK Under FF Impact*

In this section, we investigate the alteration of the Pe expression by the effect of flat-fading component, in addition to the AWGN. The Two-ray model, as shown in Fig(1.1), was found to be adequate for such analysis and is adopted here. The treatment is done in [14] for the PSK case, but we generalise this analysis to both binary and M-ary Modulation schemes. This analysis is given in Appendix II.

360 A

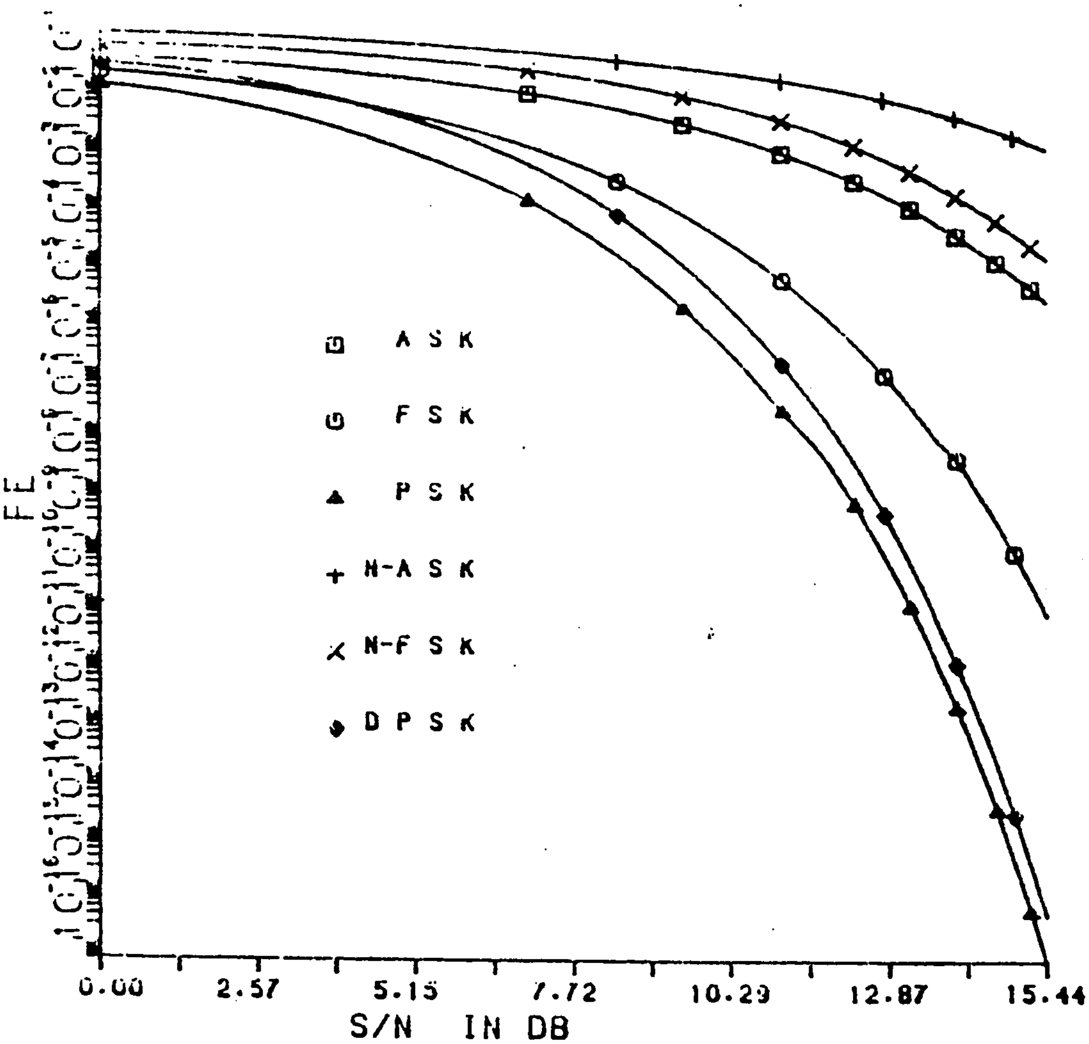


Fig 3.1 Coherent and Non-Coherent Modulation Pe Variation with S/N in presence of AWGN

3.2.2: *ASK Under FF Impact*

Proceeding with the same method, we have in case of ASK at the receiving end

$$y(t) = [Ad(t) + n_c(t)]\ cos(w_o t) + \beta d(t-\tau_m) cos w_o(t-\tau_m) - n_s(t) sin(w_o t)$$

where d(t) is the data stream of equiprobable 0 and 1

$$x(t) = y(t)\ cos(w_o t)$$

Neglecting the double frequency terms, we have

$$x(t) = \frac{1}{2}\ [Ad(t) + n_c(t)] + \frac{\beta}{2}\ Ad(t - \tau_m)\ cos(w_o \tau_m)$$

In case of Flat fading

$$d(t - \tau_m) \approx d(t)$$

$$x(t) = 1/2\ [\ Ad(t)\ (\ 1 + \beta cos\phi) + n_c(t)]$$

and

$$E(x(t)^2) = \frac{1}{8}\ A^2(1 + \beta cos\phi)^2 + \frac{E(n^2{}_c(t))}{4}$$

For the ASK case $\quad E(\ n_c(t)^2) = \eta B_T$

Hence

$$Z = S/N = \left(\frac{A^2T}{4\eta}\right)\ (1 + \beta\cos\phi)^2$$

$$= z\ (\ 1 + \beta\cos\phi)^2$$

Thus, we can write under Gaussian Noise only

$$P_e = \frac{1}{2}\ erfc\ [\sqrt{\frac{z}{2}}]$$

With the Flat Fading effect

$$P_e = [I_o(\beta z) + \beta I_1(z\beta)]\ A(z) \qquad (3.1)$$

where

$$A(z) = \frac{e^{\frac{-z}{2}}}{\sqrt{2\pi z}}$$

The reported results in Tables(4.3-5) reveals that For ASK, a 15 dB of S/N results into 0.4E-04 BER with usual Gaussian noise. An amount of β = 0.1 increases the BER to 0.8E-04 and β = 0.3 to 1.0E-03, in other words, usually, a BER of 1.0E-03, the critical BER in many digital communication systems, requires 12.9 dB. With β =0.1, it requires 13.11 dB, that is 0.21 dB more. This S/N burden is increased to 2 dB with β =0.3. For PSK, the critical BER of 1.0E-03 requires 6.9 dB, which increases to 7.15 dB with β = 0.1 and 9 dB with β =0.3, or 2.1 dB more than the usual required S/N at the receiver input.

3.3: *Flat-Fading Effects On M-ary Modulation Schemes*

In Binary Modulation schemes, each one of the two bit states is transmitted in T, the bit duration, so requiring the Nyquist bandwidth for minimum probability of bit error of

$$BW_n = \frac{f}{2} = \frac{1}{2T}$$

However, we can increase the channel capacity by reducing the required transmission BW, just by allowing one of M (M > 2) signals to be transmitted in a symbol duration

$$T_s = T \log_2 M$$

These signals are generated by changing the amplitude, phase, frequency or both the amplitude and the phase of the carrier in M discrete states to obtain an M-ary ASK, M-ary PSK, M-ary FSK or M-ary QAM " Quadrature Amplitude Modulation " schemes respectively. In these schemes, one of the M possible signal states or waveforms is assigned to a block consisting of λ binary digits where

$$\lambda = \log_2 M$$

The symbol signalling rate is then $f_s = \frac{f}{\lambda}$ and the corresponding Nyquist BW is

$$BW_s = \frac{f_s}{2} = \frac{f}{2\ \log_2 M}$$

resulting in a reduction of BW by a factor of $\log_2 M$. This conservation of transmission spectrum is unfortunately acquired at the expense of power requirement increase, complexity of signal detection and processing and mainly the increase of probability of error

In this section, our discussion is restricted to the multiphase M-PSK and combined Multiphase/Multiamplitude M-QAM signalling schemes , due to their wide use in Digital Microwaves Systems.

M-ASK is rarely used as it has proved to be inefficient in terms of amount of information per unit time, M-FSK is used in practice but when exessive transmission BW is available.

3.3.1: ***M-PSK Modulation***

In coherent M-PSK modulation [17], a phase reference must be stored at the receiver, the decision upon a transmitted waveform is taken with phase comparison between the received and the stored phases. The coherent phase detector proved to be optimum receiver in the presence of Gaussian

Noise.

Let the received waveform

$$z_{k(t)} = A\ cos(w_c t + \phi_k) + n(t) \tag{3.2}$$

where ϕ_k is one of the possible M phases and

$$\phi_k = \frac{2\pi k}{M} \quad ; \qquad k = 1,2,\ldots\ldots M$$

$$z_k(t) = \{A + n_c(t)\}\ cos(w_c t + \phi_k) - n_s(t)\ sin(w_c t + \phi_k)$$

and phase $z_k(t) = \phi_k + \theta$

where

$$\theta = \frac{\tan^{-1} n_{s(t)}}{A + n_c(t)}$$

The error is committed whenever the phase mesurement device decides on a phase laying outside the interval

$$\phi_k - \frac{\pi}{M} < \theta < \phi_k + \frac{\pi}{M}$$

The pdf of the phase has a well-known expression

$$p(\theta) = \frac{1}{2\pi}\ e^{-2z}\ [1 + \sqrt{8\pi z}\ \cos\theta\ e^{2z\cos^2\theta}\ Q(R)] \tag{3.3}$$

$$-\pi < \theta < \pi$$

where $$R = \sqrt{4z}\cos\theta$$

and $$Q(x) = \frac{1}{2\pi}\int_{x}^{\infty} e^{\frac{-x}{2}}\, dx$$

$$= \frac{1}{2}\, erfc\sqrt{\frac{x}{2}}$$

z is the S/N ratio at the input of the receiver given by

$$z = \frac{A^2 Ts}{2\eta}\ , \text{ and } Ts = T\log_2 M$$

Finally, the probability of symbol error has the following expression

$$Pe = 1 - \int_{-\frac{\pi}{M}}^{\frac{\pi}{M}} p(\theta)\, d\theta \qquad (3.4)$$

For coherent 2, and 4 PSK the probability has the closed form

$$Pe_2 = Q[\sqrt{2z}]$$

$$= \frac{1}{2}\, erfc(z)$$

$$Pe_4 = 1 - [1 - Q(\sqrt{z})]$$

For high S/N ratio, the M-PSK Pe expression can be reduced to

$$Pe \simeq 2\, Q\left[\sqrt{\frac{A^2 Ts}{\eta} \sin^2\frac{\pi}{M}}\right] \tag{3.5}$$

This is depicted in Fig(3.2)

Before investigating the FF effect on M-ary PSK, we present the demodulation scheme at the receiving end. The M-PSK signal can be written as

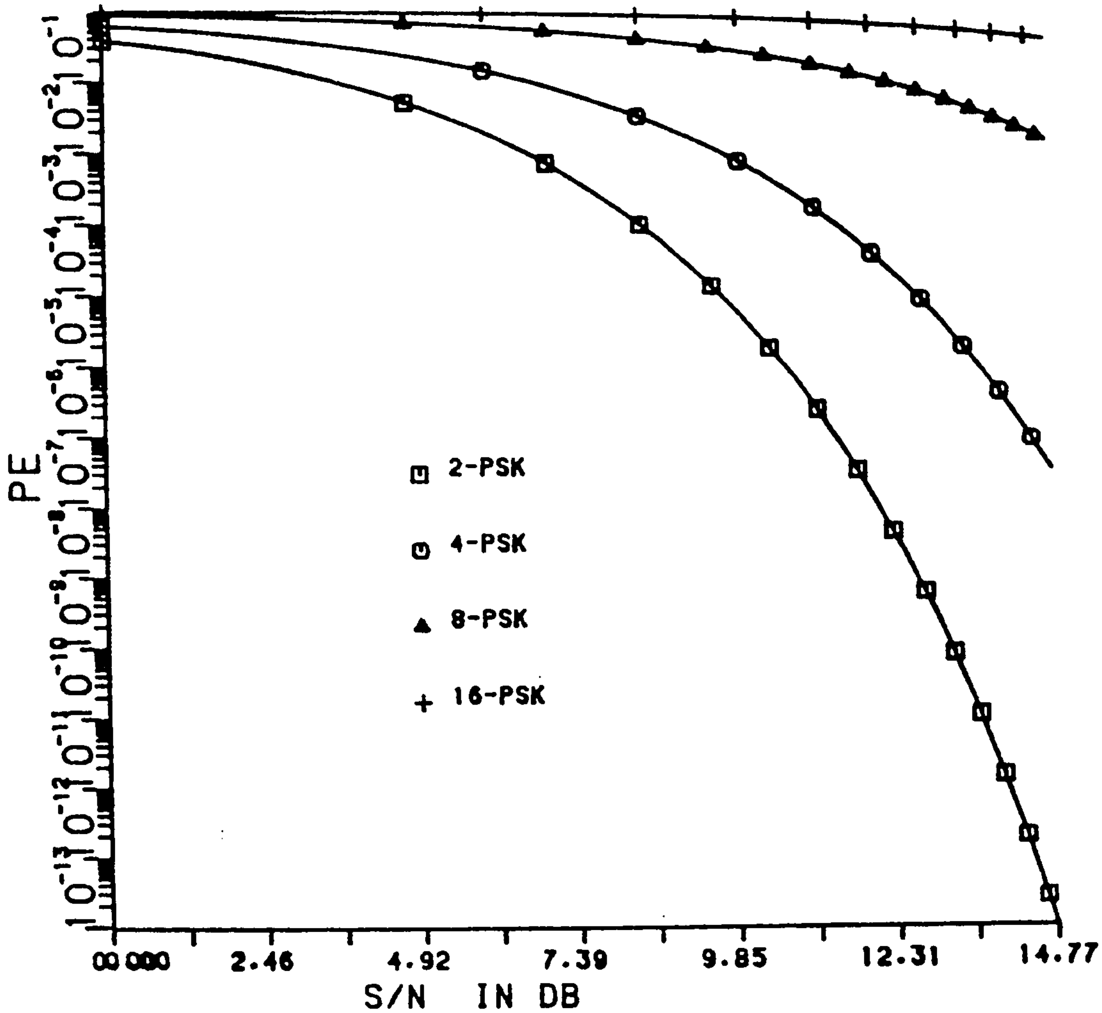
$$z(t) = A \sum_{k=0}^{\infty} g(t - kTs)\, cos(w_c t + \phi_k) \tag{3.6}$$

where g(t) is the Nyquist BB shaping signal to yield zero ISI, usually taken as the raised cosine shape at the transmitting end, and $\{\phi_k\}$ carries the digital information,

$$\phi_k = 2\pi\frac{k}{M} \qquad k = 1,2,\ldots M$$

$$z(t) = A\, cosw_c t \sum_{k=0}^{M} \cos\phi_k\, g(t - kTs)$$

$$- A\ sinw_c t \sum_{k=0}^{M} \sin\phi_k\, g(t - kTs)$$



Fig(3.2):M-PSK Modulation Pe Variation With S/N in presence of AWGN

Which shows that z(t) is a superposition of two streams of BB signals weighted by

$$a_k = \cos\phi_k \text{ and } b_k = \sin\phi_k$$

in quadrature, this form of z(t) offers a demodulation scheme as shown in Fig(3.3) for 4-PSK, as an example.

Let consider the QPSK or 4-PSK , the signal states are

$$s_1(t) = A\cos w_c t \text{ --------> } +1\ +1$$

$$s_2(t) = -A\sin w_c t \text{ --------> } -1\ -1$$

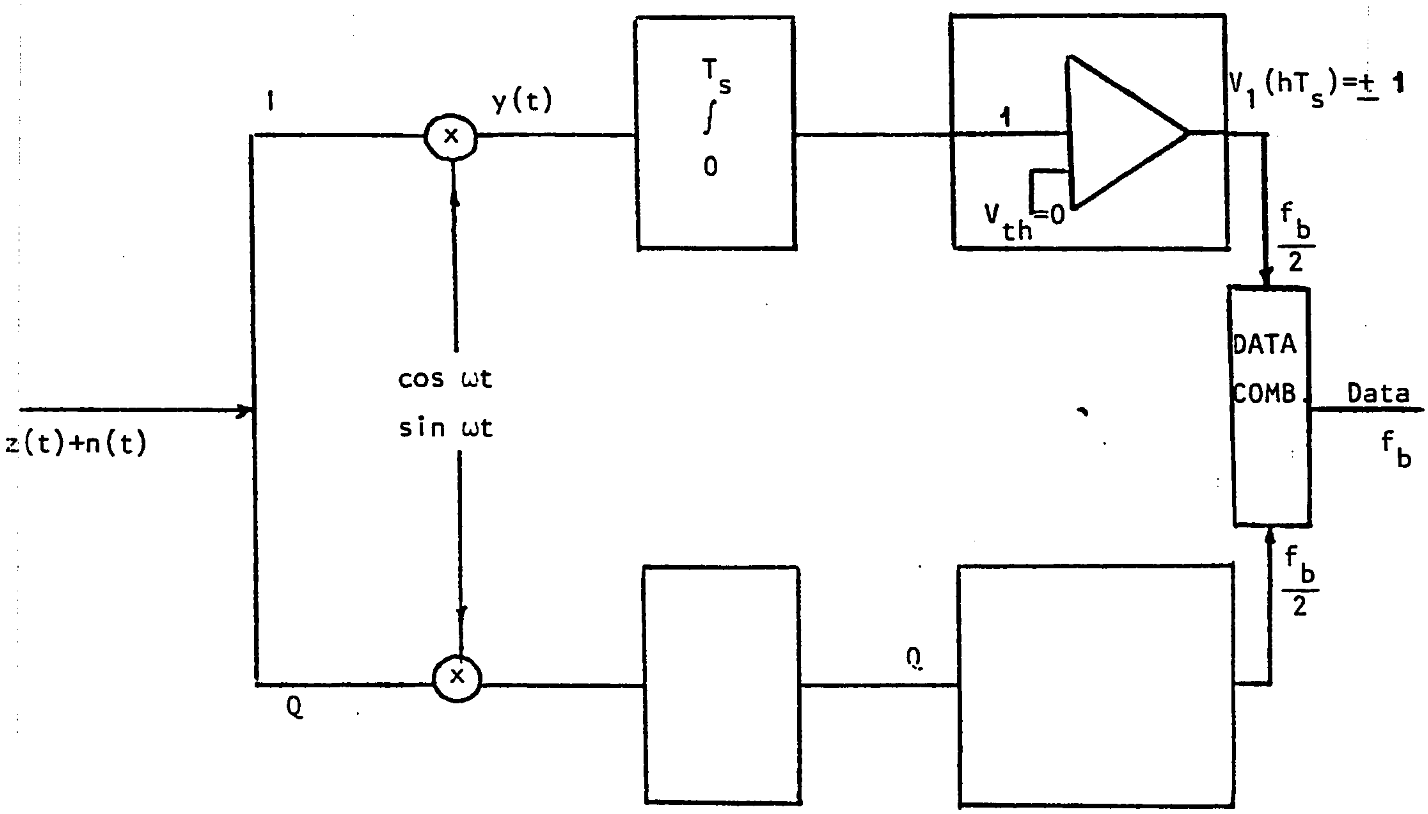
$$s_3(t) = A\sin w_c t \text{ --------> } -1\ +1$$

$$s_4(t) = -A\cos w_c t \text{ --------> } +1\ -1$$

for $0 < t < Ts$

These waveforms correspond to the phase shifts of 0^o, 90^o, 180^o, and 270^o.

for M = 4 , $L = \log_2 M = 2$



Fig(3.3):4-PSK or 4-QAM coherent demodulator

The comparator will measure the input and then generate a positive bit for +A and a negative one for -A, and then decides on the signal transmitted.

For example, that if comparator 1 has generated a positive bit and also has done comparator 2, the decision is for $s_1(t)$.

The task now is to find the probability of error when a stream of data of the form

$$y(t) = \sum_{k=0}^{M} \cos\phi_k \, g(t - kTs)$$

is being detected by the logic of comparators.

As in binary PSK, we have considered that the affected S/N

$$z' = z\,(1 + \beta\cos\phi)^2$$

we can consider the expression of Pe for the M-ary PSK to be

$$Pe/_{\phi} = 1 - \frac{1}{2\pi} \int_{-\frac{\pi}{M}}^{\frac{\pi}{M}} e^{-2z'} \, k(\theta,\phi)\, d\theta \qquad (3.7)$$

where

$$k(\theta,phi) = 1 + \sqrt{8\pi z'}\, \cos\theta \; e^{2z'\cos^2\theta} \; Q(R)$$

and $R = \sqrt{4z'} \cos\theta$, $z' = z\,(1 + \beta\cos\phi)^2$

Finally, the probability of symbol error has the following expression

$$Pe = \frac{1}{2\pi} \int_{-\pi}^{\pi} Pe/\phi \; d\phi \tag{3.8}$$

The M-PSK Pe expressions are evaluated and shown in Fig(3.4), the dramatic effect of FF can be clearly seen.

3.3.2: M-QAM Modulation

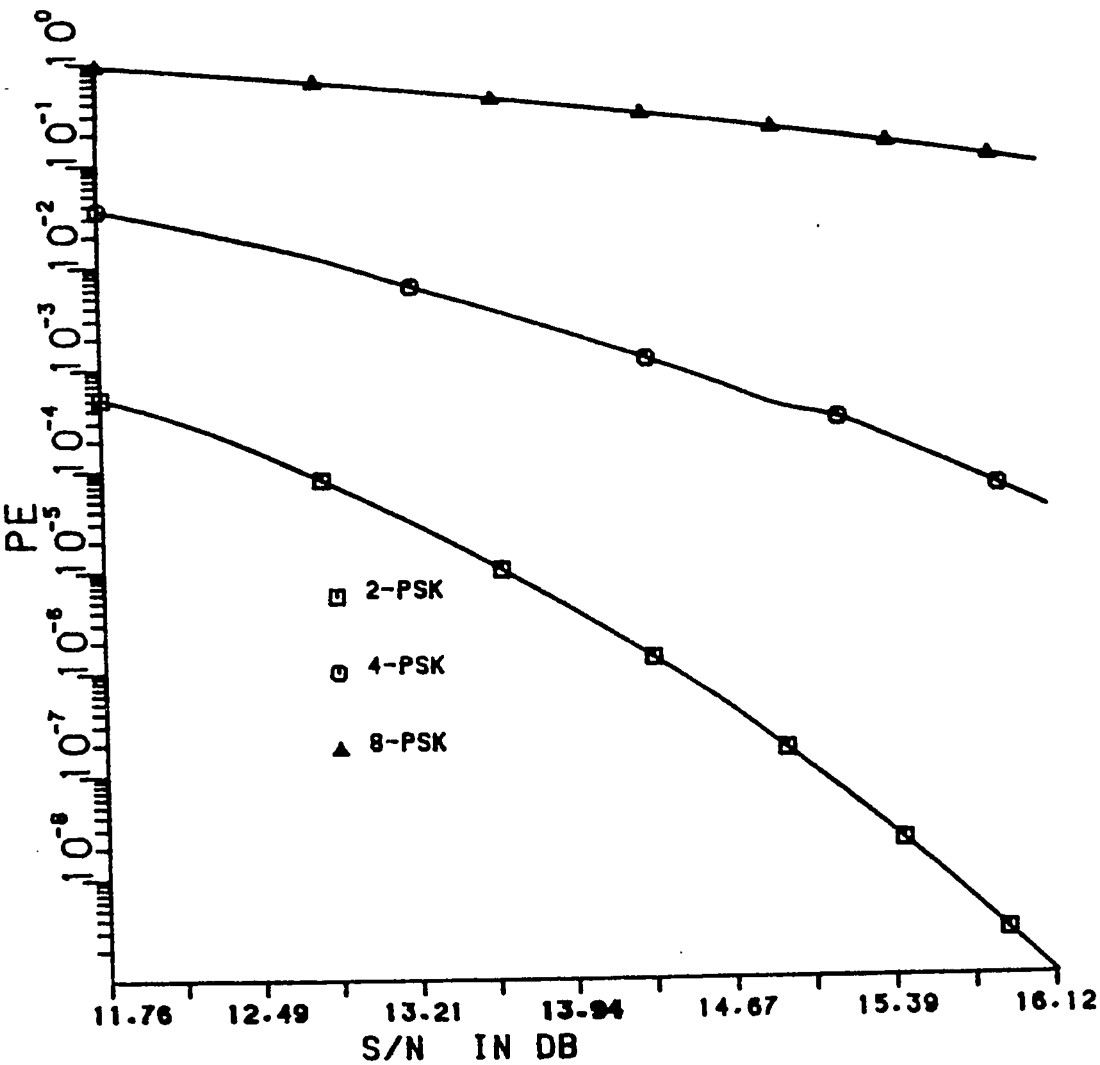
The new trend in digital communication systems to increase the channel capacity for high-speed data transmission, suggests the use of high-level modulation schemes. M-QAM offers the best trade-off between the theoritical performance and implementation complexity.

The 16-QAM technique has found wide use in recent high-capacity digital microwave systems and proved to be efficient

The modulated signal can be represented by

$$z(t) = \sum_{k=0}^{\infty} [a_k \cos(w_c t) - b_k \sin(w_c t)]\; g(t - kTs) \tag{3.9}$$

a_k and b_k are multilevel random variables and independent given by



Fig(3.4):M-PSK Modulation Pe Variation With S/N in presence of Flat-Fading (Beta = 0.3)

$$(a_k, b_k) = [\pm a, \pm 3a, \pm 5a, \ldots\ldots, \pm(\sqrt{M}-1)a]$$

where $M = 4^k$, $k = 1,2,3 \ldots.$

The signal average power for the level spacing at the receiver input is given by

$$E[a^2{}_k] = \frac{M-1}{3}\, a^2 \tag{3.10}$$

The signal constellation for the 16-QAM is depicted in Fig(3.5).

As the M-ary PSK, M-QAM consist of two multilevel AM signals in quadrature, the main difference is that all the waveforms in M-PSK have the same amplitude, but in M-QAM, every signal state has its own amplitude and phase.

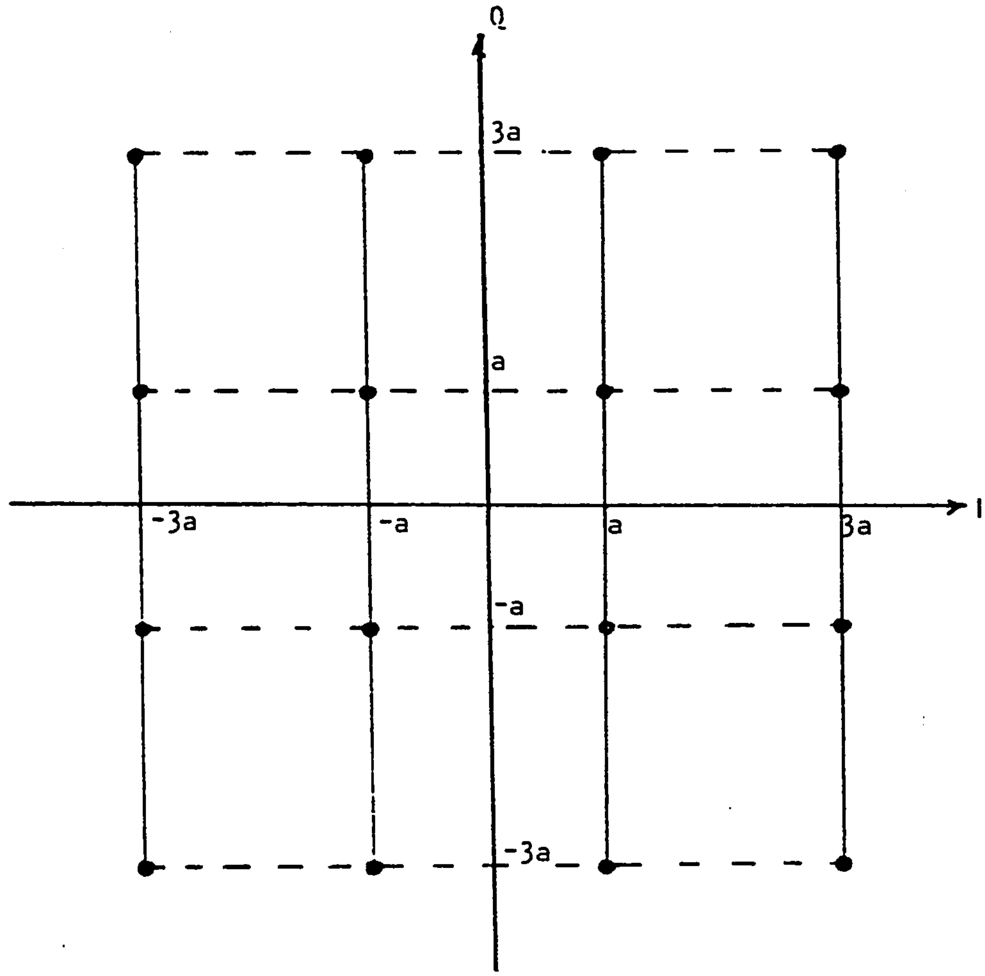
the demodulator is identical to the one used for M-PSK, the only difference is in the threshold levels.

The signal at the inphase channel is given by

$$z_i(t) = \sum_{k=0}^{\infty} a_k\, g(t - kTs)\, \cos w_c t + n(t)$$

Then after the removal of the carrier frequency by the LPF

$$y_i(t) = \frac{1}{2} \sum_{k=0}^{\infty} a_k\, Ts\, g(t - kTs) + N$$



Fig(3. 5):16-QAM constellation

where $$N = \int_0^{ts} n(t)\ cosw_c t$$

$$y_i(mTs) = \frac{1}{2}\ a_m Ts + \sum_{\substack{k=0 \\ k!=m}}^{\infty} \frac{a_k}{2}\ Ts\ g((m-k)Ts) + N$$

where the first element is the desired m^{th} transmitted symbol and the others are ISI and noise term respectively,

With g(t) raised cosine function, the value of ISI is zero, then we have only

$$y_i(mTs) = \frac{1}{2}\ a_m Ts + N$$

$$E(N) = 0 \quad \text{and } E\ (N^2) = \frac{\eta Ts}{4}$$

The probability of error for this output can be evaluated as shown in the following systems.

Let the 16-QAM, be characterized by the following levels

$$(a_{a,}\ b_k) = [\pm a,\ \pm 3a]$$

The strategy of detection and decision is as follos:

$$Y = V + N$$

if $Y > 2aTs$, $Yc = 11$

if $0 < Y < 2aTs$, $Yc = 10$

if $-2aTs < Y < 0$, $Yc = 01$

if $Y < -2aTs$, $Yc = 00$

$$V = V(kTs) = \int_0^{Ts} a_k \cos w_c t \cos w_c t \, dt$$

$$= \frac{a_k Ts}{2}$$

and

$$N = \int_0^{ts} n(t) \cos w_c t \, dt$$

so $E(N) = 0$, and $\sigma^2_n = E(N^2) = \frac{\eta Ts}{4}$

An error occurs whenever the sampled level is not in the appropriate decision interval, the Pe per channel is then

$$Pel = 1/4 \, [\, P(E/00) + P(E/10) + P(E/01) + P(E/11)]$$

this is based on the assumption that the per-channel 2-bits are equiprobable, and the symbol-level correspondance is as following:

+3a ------> 11

+1a ------> 10

-1a ------> 01

-3a ------> 00

$$P(E/11) = P[\,Y{+}{+} < 2aTs\,]$$
$$= P[\,V{+}{+} + N < 2aTs\,]$$
$$= P[\,3aTs + N < 2aTs\,]$$

we have not considered the half term for clear demonstration, but it is similar.

$$P(E/11) = \frac{1}{\sqrt{2\pi\sigma^2_n}} \int_{-aTs}^{aTs} e^{\frac{-n^2}{2\sigma^2_n}}\, dn$$

$$= \frac{1}{2}\, erfc[\frac{a^2Ts}{2\eta}]$$

$$P(E/10) = P[\,Y{+}{-} > 2aTs\,] + P[\,Y{+}{-} < 0\,]$$
$$= P[\,V{+}{-} + N > 2aTs\,] + P[\,V{+}{-} + N < 0\,]$$
$$= P[\,aTs + N > 2aTs\,] + P[\,aTs + N < 0\,]$$
$$= P[\,V{+}{-} + N > 2aTs\,]$$
$$= \mathrm{erfc}\,[\frac{a^2Ts}{2\eta}]$$

as $P(E/11) = P(E/00)$, and $P(E/01) = P(E/10)$

$$Pe1 = \frac{3}{4}\, erfc\,[\frac{a^2Ts}{2\eta}]$$

where $Ts = T \log_2(M)$

considering the quadrature channel, which can be seen as an uncorrelated channel with the inphase one. The overall probability is finally

$$Pc = (1 - Pe_1)\ (1 - Pe_2)$$

so $$Pe = 1 - Pc \approx 2Pe1 - Pe1^2$$

This result can be generalized for any M-QAM by

$$Pe1 = \frac{L-1}{L}\ erfc[\frac{a^2 Ts}{\eta}]$$

where $$Ts = T \quad \log_2(M) \ , \text{ and } L = \sqrt{M}$$

in term of signal power

$$Pav = E\ (a^2{}_k) = (\frac{M-1}{3})^2$$

$$Pe1 = \frac{L-1}{L}\ erfc\ (\frac{3z}{\sqrt{M-1}})$$

where $$z = \frac{PavTs}{2\eta} \ , \ Ts = T \quad \log_2(M)$$

and $$Pe = 2Pe1 - Pe1^2$$

Table(4.13) shows the Pe values for M-QAM with S/N and Table(4.6) shows the superiority of 16-QAM over 16-PSK

With Falt Fading, we get directly

$$Pe1/\ \phi = \frac{L-1}{L}\ erfc\ (\frac{3z'}{\sqrt{M-1}})$$

where $z' = z\,(1 + \beta\cos\phi)^2$

Then finally

$$Pe = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{L-1}{L}\ erfc\left(\sqrt{\frac{3z'}{(M-1)}}\right)d\phi$$

These derivations are in fact valid for M-PAM in quadrature, for M-QAM, we have to consider one half in the signal term to get

$$Pe = \frac{1}{2\pi}\int_{-pi}^{\pi}\frac{L-1}{L}\ erfc\left(\sqrt{\left(\frac{3z'}{2(M-1)}\right)}\right)d\phi$$

The impact of FF on 4-QAM are illustrated in Fig(3.6).

3.4: Frequency-Selective Fading Effect On M-QAM

3.4.1: Frequency-Selective Fading Impact On 4-QAM

When $0 < \frac{\tau_m}{T} < 1$, the successive bits d(t) and $d(t - \tau_m)$ overlap, causing ISI. This is illustrated in Fig(3.7).

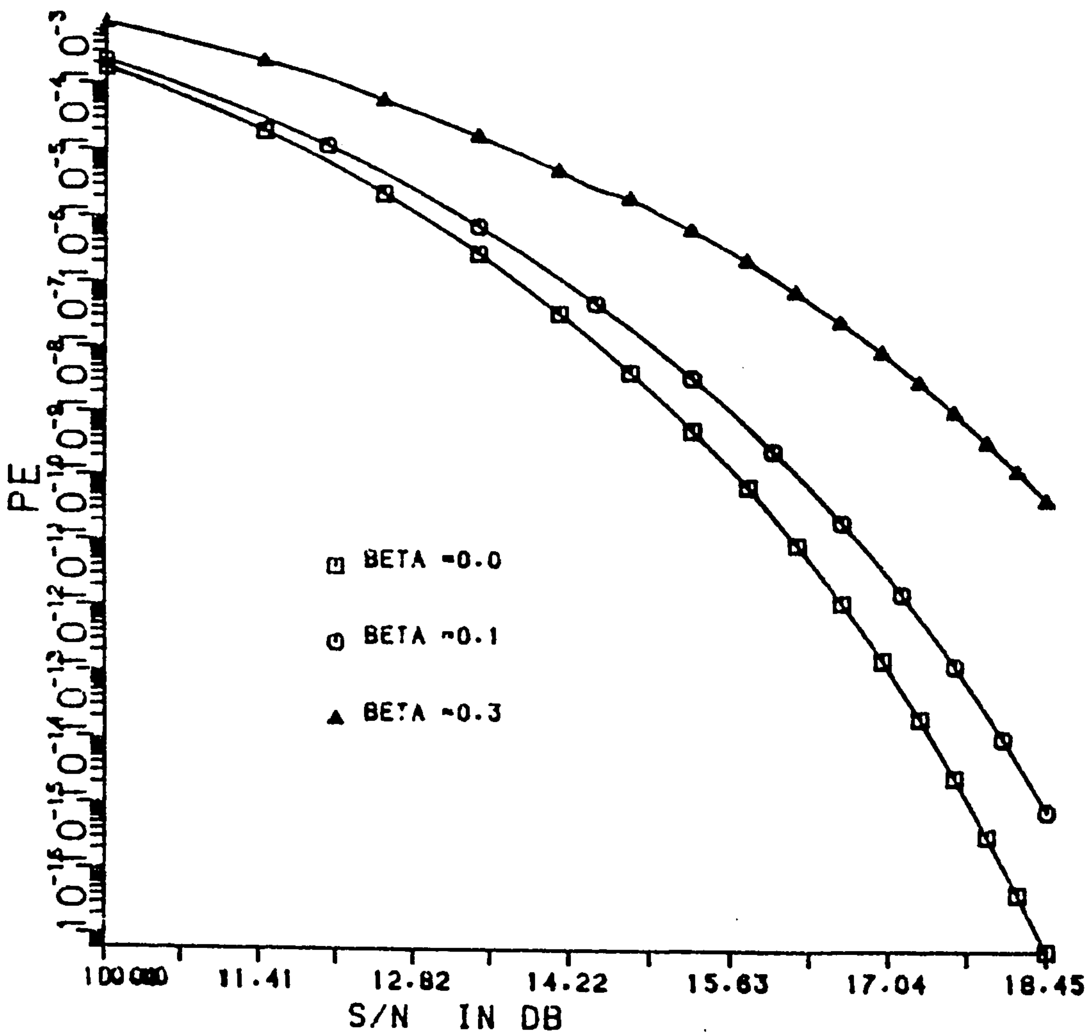
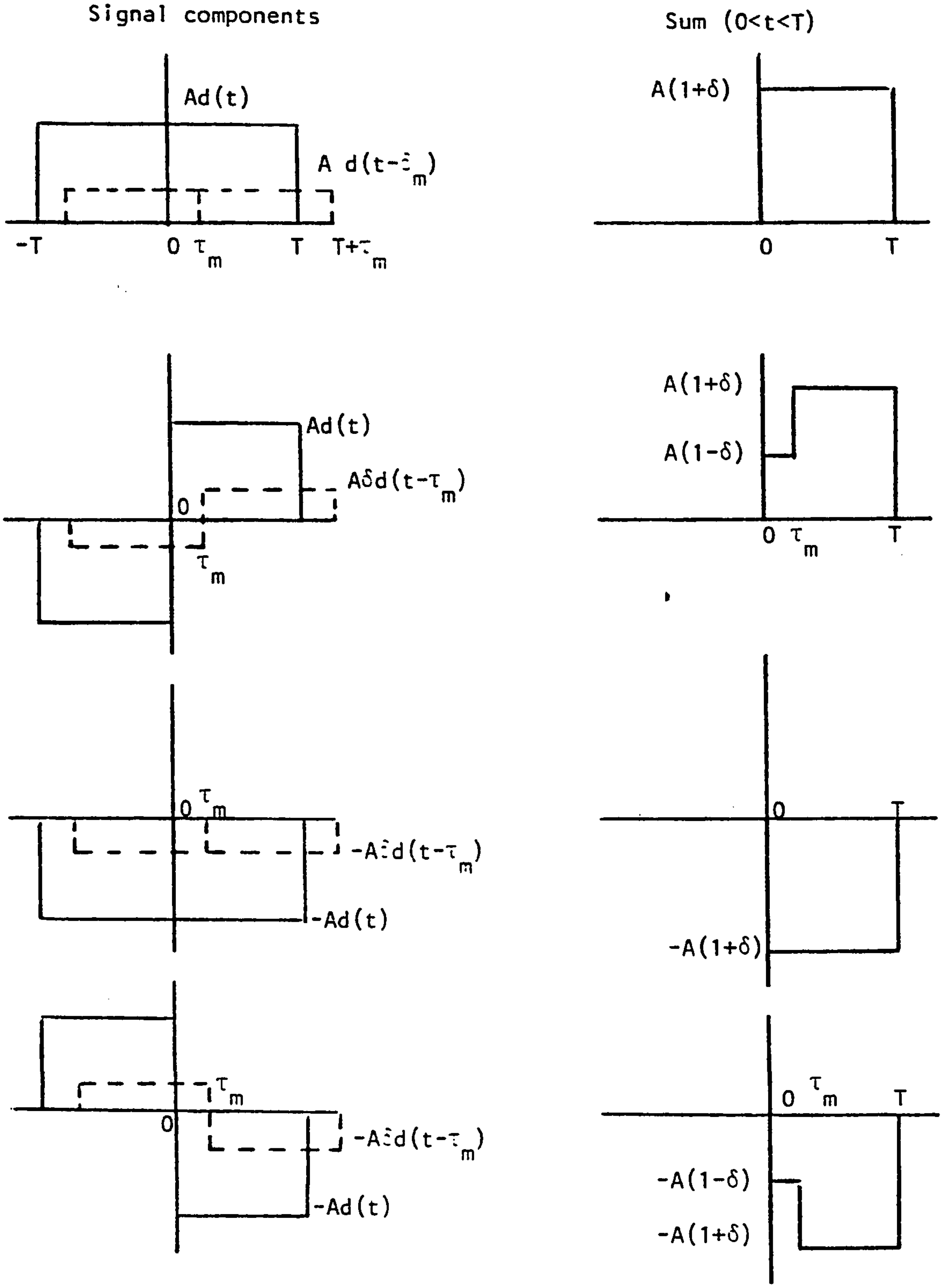


Fig 3.6 4-QAM Pe evaluation with S/N in presence of Flat-Fading



Fig(3.7):The possible states of errors in PSK due to 2 bits

The received signal is

$$y(t) = s(t) + \beta s(t - \tau_m) + n(t)$$

For PSK case, $s(t) = Ad(t) \cos(w_o t)$, $d(t) = \pm 1$

$$x(t) = A\, d(t) + \delta\, A\, d(t - \tau_m) + n_c(t) \qquad (3.12)$$

with $\delta = \beta \cos(w_o \tau m)$

Considering that the transmitted bits, zeros and ones, are equiprobable, and taking the overlapping effect of two successive bits only, we have four possible states as illustrated in the Fig (3.7)

$$Pe = \frac{1}{4} [P(E/11) + P(E/10) + P(E/01) + P(E/00)]$$

due to the noise pdf and signal symmetries, we have

$$P(E/11) = P(E/00)$$
$$P(E/10) = P(E/01)$$

the signal component at the output's integrator, given that 11 were transmitted, is

$$V{++} = AT(1 + \delta)$$

while if 01 were transmitted,

$$V{-+} = AT(1 + \delta) - 2A\delta\tau_m$$

$$= AT\{(1 + \delta) - 2A\delta(\frac{\tau m}{T})\}$$

The output of the integrator is given by

$$Y = V \; + N$$

where V refers to the signal component and N to the noise component, determined by

$$N = \int_{0}^{ts} 2\, n(t)\, cos(w_o t)dt$$

$$E(N) = 0 \quad \text{and} \quad \sigma^2_N = E(N^2) = \eta T$$

$$\begin{aligned} P(\,E/11) &= P[\; Y < 0 \;] \\ &= P[\; V{+}{+} \; + N < 0 \;] \\ &= P[\; N < - \; V{+}{+} \;] \\ &= \frac{1}{2}\, erfc\{\sqrt{z}(1 + \delta)\} \end{aligned}$$

where

$$z = \frac{A^2 T}{2\eta}$$

$$\begin{aligned} P(\,E/01) &= P[\; Y < 0 \;] \\ &= P[\; V{-}{+} \; + N < 0 \;] \\ &= P[\; N < - \; V{-}{+} \;] \\ &= \frac{1}{2}\, erfc\{\sqrt{z}(1 + \delta - 2\delta\frac{\tau_m}{T})\} \end{aligned}$$

thus the probability of error is given by

$$Pe = \frac{1}{4}\, erfc\{\sqrt{z}(1 + \delta)\}$$

$$+ \frac{1}{4}\, erfc\{\sqrt{z(1 + \delta - 2\delta\frac{\tau_m}{T})}\} \qquad (3.13)$$

For non Multipath-Fading, $\delta =$ and $\frac{\tau_m}{T} = 0$

and $$Pe = \frac{1}{2}\, erfc\, \sqrt{z}$$

In 4-QAM or QPSK system, the coherent receiver is composed of two binary phase detectors in quadrature,the Pe expression per channel is similar to that presented previously.

the probability of correct detection is

$$Pc = (1 - Pe1)(1 - Pe2)$$

The two binary channels are statistically independent due to the presence of gaussian noise.

$$Pe1 = Pe2, \text{ and } Pe = 1 - Pc \simeq 2\, Pe1$$

Hence, $$Pe \simeq \frac{1}{2}\, erfc\{\sqrt{z}(1+\delta)\}$$

$$+ \frac{1}{2}\, erfc\{\sqrt{z(1 + \delta - 2\delta\frac{\tau_m}{Ts})}\} \qquad (3.14)$$

3.4.2: Frequency Selective Fading Impact on 16-QAM

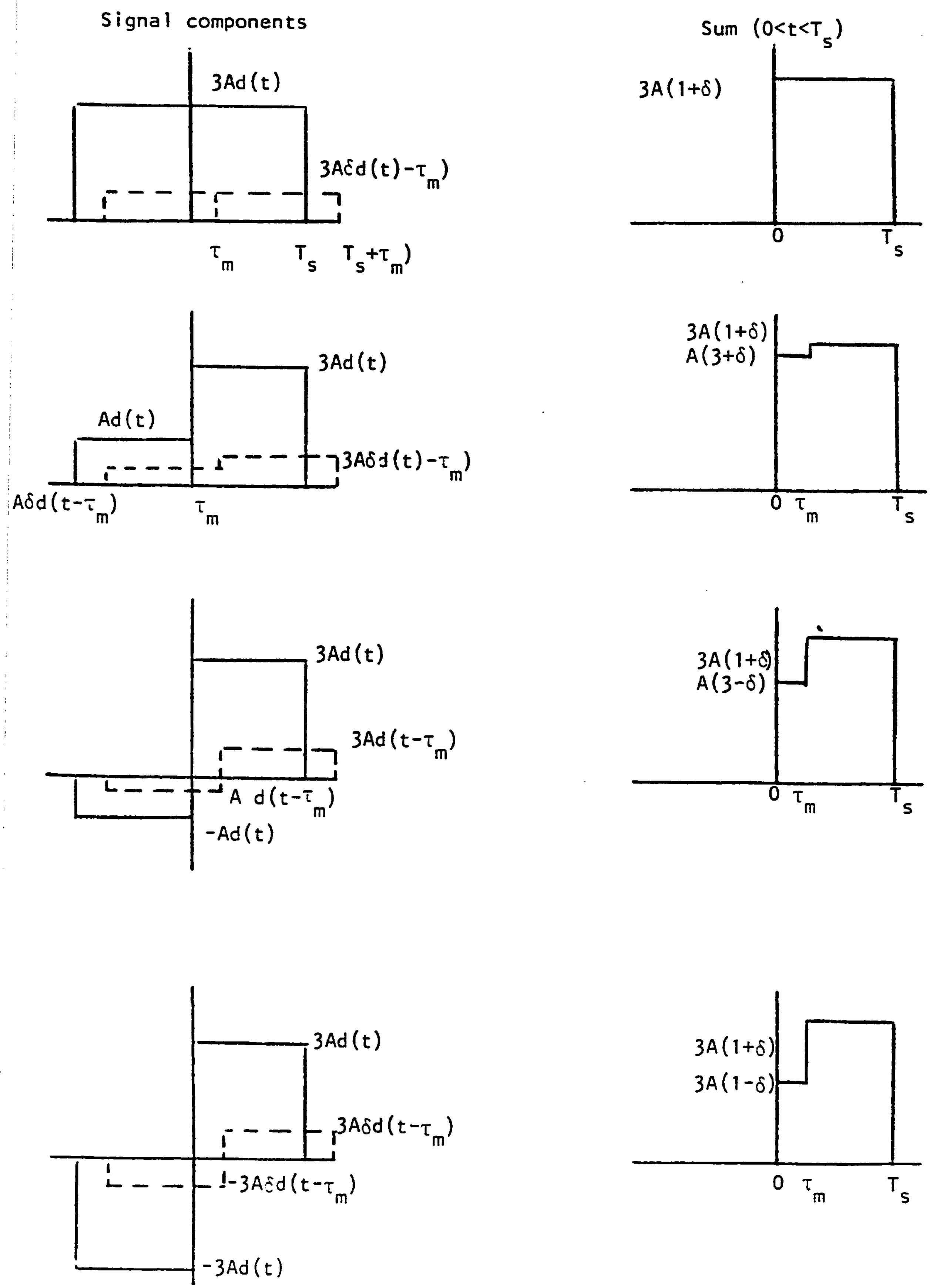
The 16-QAM digital radio system is becoming, at present, widely used due to its high spectral efficiency and high-speed data transmission. The study of the FSF impact on it is very important in order to evaluate the system performance. The 16-QAM conFiguration consists of 2^4 or 16 states. By following similar procedure to the one used to derive the QPSK or 4-QAM Pe expressions under the FSF effects, the 16-QAM expressions are obtained.

Due to the similarity between the two channels in quadrature of the modulation, the treatment is done only for one channel.

From Fig(3.8), we can say that:

$$\begin{aligned} Pe1 = 2/16\ [&P(E/0011) + P(E/1011) + P(E/0111) + P(E/1111) + \\ &P(E/0001) + P(E/0101) + P(E/1001) + P(E/1101)\ ] \end{aligned}$$

The complete derivation is reported in Appendix III.



Fig(3.8): The possible states of errors in 16-QAM when the first symbol is 11

Generally, the M-QAM probability expression can be found by:

$$Pe1 = \frac{2}{M}\Big[\frac{1}{2}\sum_{k=0}^{m}\Big[\sum_{\substack{i=-(m-1)\\ i\neq 0}}^{(m-1)} erfc\, z'[1 + l_i\delta + l_i(1-k)\delta(\frac{\tau}{Ts})]$$

$$+ erfc\, z'[1 + l_m\delta - k\delta(\frac{\tau}{Ts})]\Big]\Big] \qquad (3.15)$$

where $m = \sqrt{M} - 1$

$$z' = \sqrt{\frac{3z}{2(M-1)}}$$

z : the average symbol S/N, and $Ts = T \log_2(M)$

and l_i belongs to the pair (a_n , b_n)

Finally, we evaluate the probability by

$$Pe \approx 2Pe1 - Pe1^2$$

As β and τ are random variables, the last evaluated Pe is conditional. Let $f_\beta(\beta)$ and $g_\tau(\tau)$, the pdf of β and τ respectively, the M-QAM Pe is given by

$$Pe = \iint_{\beta\tau} Pe/_{(\beta,\tau)}\, f_\beta(\beta)\, g_\tau(\tau)\, d\beta d\tau \qquad (3.16)$$

A set of Figures has been produced to understand the impact of the different parameters β, δ, and τ The 4-QAM scheme has been studied with the term δ in the Figs(3.9-13), and with the term $\frac{\tau}{Ts}$ in the Figs(3.14-17). The 16-QAM has, similarly, studied with the parameter β only to examine its effects on Pe with $\frac{\tau}{Ts}$ This is illustrated in the Figs(3.18-20), and for small increments of τTs in Figs(3.21-26). The work is extended by considering 3 S/N level and evaluating the behavior of Pe with β, and $\frac{\tau}{Ts}$, for foT = 3. The result is illustrated in Figs(3.27-32), and for foT = 100 in Fig(3.33).

3.4.3: Results Analysis And Interpretation

The parameter $\delta = \beta\ cos(w_o\tau)$ plays an important role in deterioting or enhancing Pe values, that is, it contributes with destructive or constructive effects on the system performance. For negative δ , the system suffers from very high BER, even though, it is noticed that for a fixed negative δ , P decreases as the delay fraction increases. Fig(3.9) shows this fact for 4-QAM with δ = -0.7, however, for high S/N, the effect of delay fraction becomes negligible. The overall behaviour is kept similar as the values of δ increases to -0.2, but, the probability of error decreases dramatically approaching the system performance with gaussian noise. This is clear in table (4.16) and Fig(3.11). As δ acquires positive values, which means a phase $|w_o\tau| < \frac{\pi}{2}$

, the effect of FSF becomes constructive and Pe values are ameliorated beyond Gaussian conditions. However, the delay fraction has a distinguishable role in deteriorating the system performance, as illustrated by Figs(3.12-13). When fixing the delay, the parameter δ impact is seen clearly in Fig(3.14). Increasing the delay has no big effect on Pe for $\delta < 0$, however, a dramatic change occur for $\delta > 0$. With δ = 0.2, $\frac{\tau}{T}$ = 0.1, a 10.4 dB results in 1.0E-04 BER, increases to 0.3E-03 with $\frac{\tau}{T}$ = 0.4 and to 0.4E-02 with $\frac{\tau}{T}$ = 1.0, Figs(3.14-17) illustrate well this remark.

The understanding of the parameter roles can be better by varying only β and $\frac{\tau}{T}$ This is done with 16-QAM case. For β = 0.1, and as $\frac{\tau}{T}$ increases, one can notice a drop in Pe as approaches 0.2 and a maximum near 0.5. As β increases, Fig(3.20) illustrates clearly that only for $\frac{\tau}{T}$ = 0.2 has the lowest value of Pe, and the others $\frac{\tau}{T}$ has nearly similar values. Table(4.25) shows that a S/N of 17.9 dB with β =1.0 results in a Pe of 0.23E-01 with $\frac{\tau}{T}$ = 0.2, and in nearly 0.75 with other values of $\frac{\tau}{T}$ Fig(3.21-26) depict this behaviour with smaller increments of $\frac{\tau}{T}$.

The picture is more clear when taking 3 S/N levels 10.4 dB, 13.2 dB and 17.9 dB, and examing Pe variation with β for specified values of $\frac{\tau}{T}$ or vise-versa. The overall behaviour, which is illustrated in Figs(3.27-33), is in fact similar the voltage or power attenuation channel transfer function reported in Fig(2.3-4). Besides, Fig(3.30) and Fig(3.33) reveal that as f o T increases, that is the carrier frequency or the bit duration increases, the Pe variation has a decay, This emphazises the need to increase the carrier frequency to combat FSF effects.

The FSF impact on the system performance is better understood by finding the average probability that the resultant Pe exceeds a threshold one. The conditionnal Pe on β and τ has been evaluated by considering the pdf of β and τ as in the Two-ray model

The outage for M-QAM digital modulation schemes under FSF is given by

$$\text{Outage} = \int_{\beta 1}^{\beta 2} \int_{\tau 1}^{\tau 2} Pe/_{\beta,\tau} \, p_{\beta,\tau} \, (\beta,\tau) \, d\beta \, d\tau$$

Where (β1,β2),(τ1,τ2) : the pair intervals which cause the system to be In outage.

$Pe/_{\beta,\tau}$: the conditionnal Pe for M-QAM

$p_{\beta,\tau}(\beta,\tau)$: the pdf of β and τ given by

$$p_{\beta,\tau}(\beta,\tau) = p_\beta(\beta)\ p_\tau(\tau)$$

because β and τ are independent R.V.

The pdf of β and τ are given by [16]

$p_\beta(\beta)$: uniformly distributed in [0,1]

$$p_\tau(\tau) = (\tau/\tau_o) \exp(-\tau/\tau_o)\ u(\tau)$$

where $\tau_o = E(\tau)$

The outage is evaluated and the results are reported in APP-V and illustrated in Figs(5.8-9)

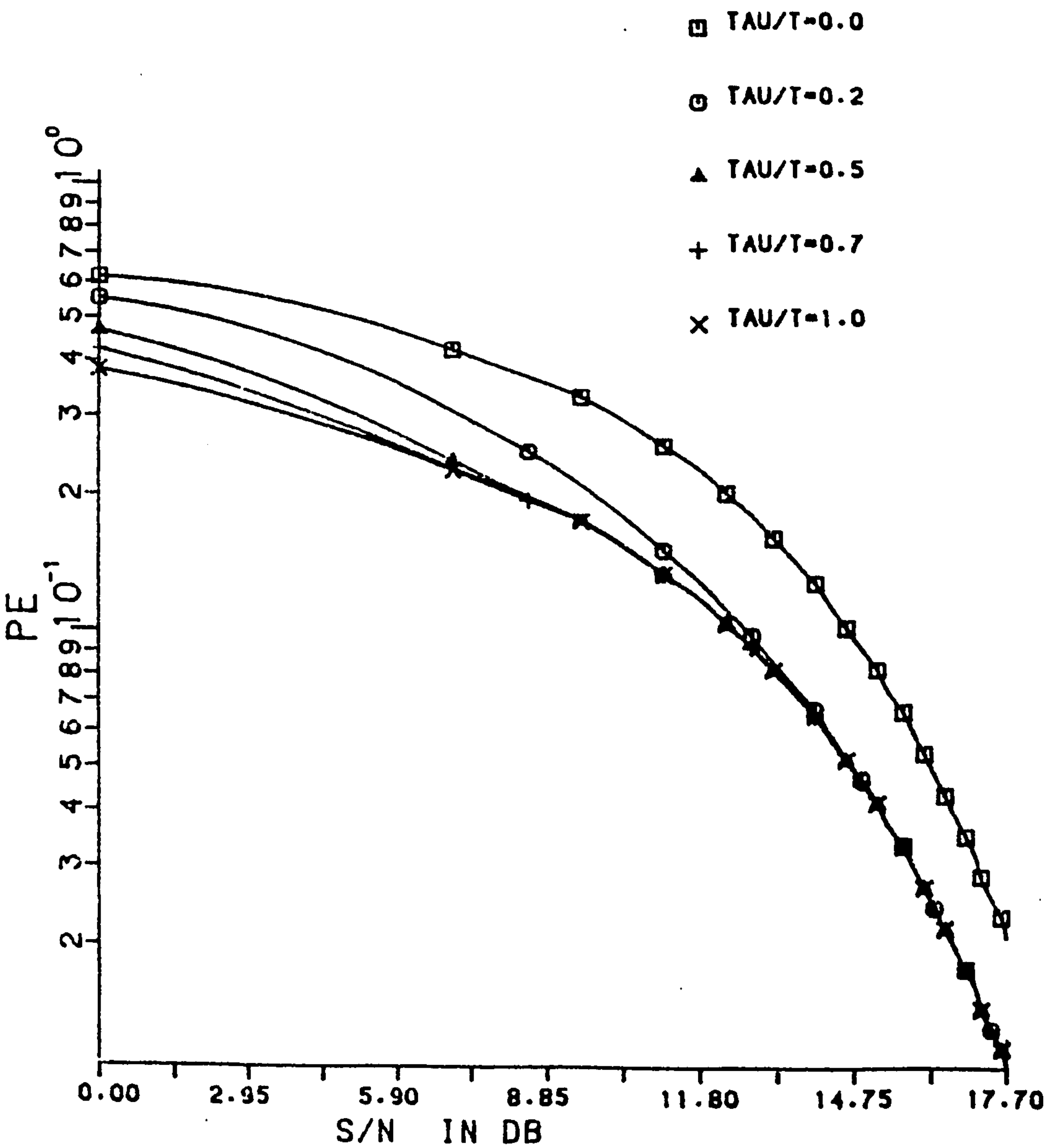


Fig 3.9 4-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (Delta = -0.7)

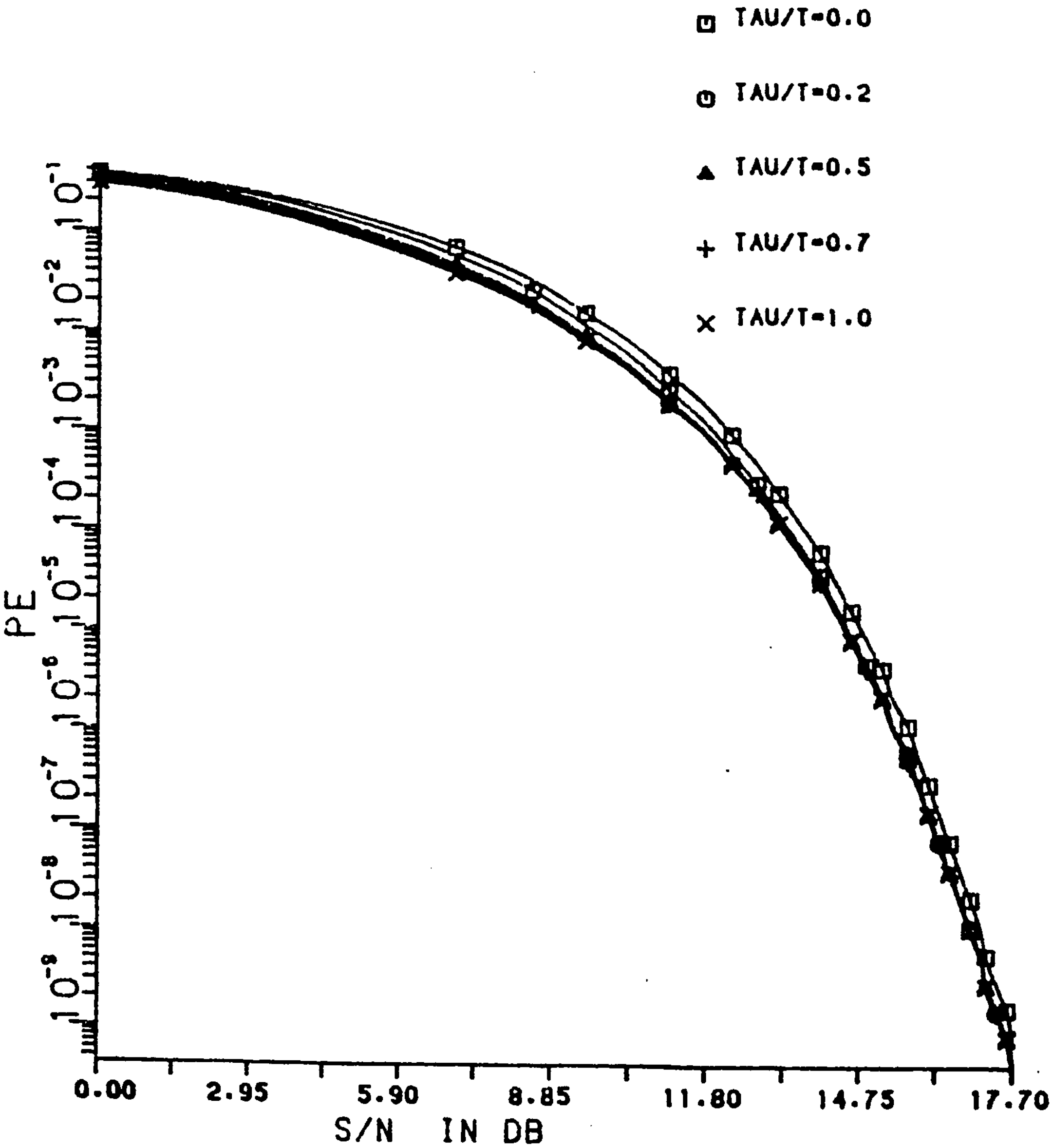


Fig 3.10 4-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (Delta = -0.2)

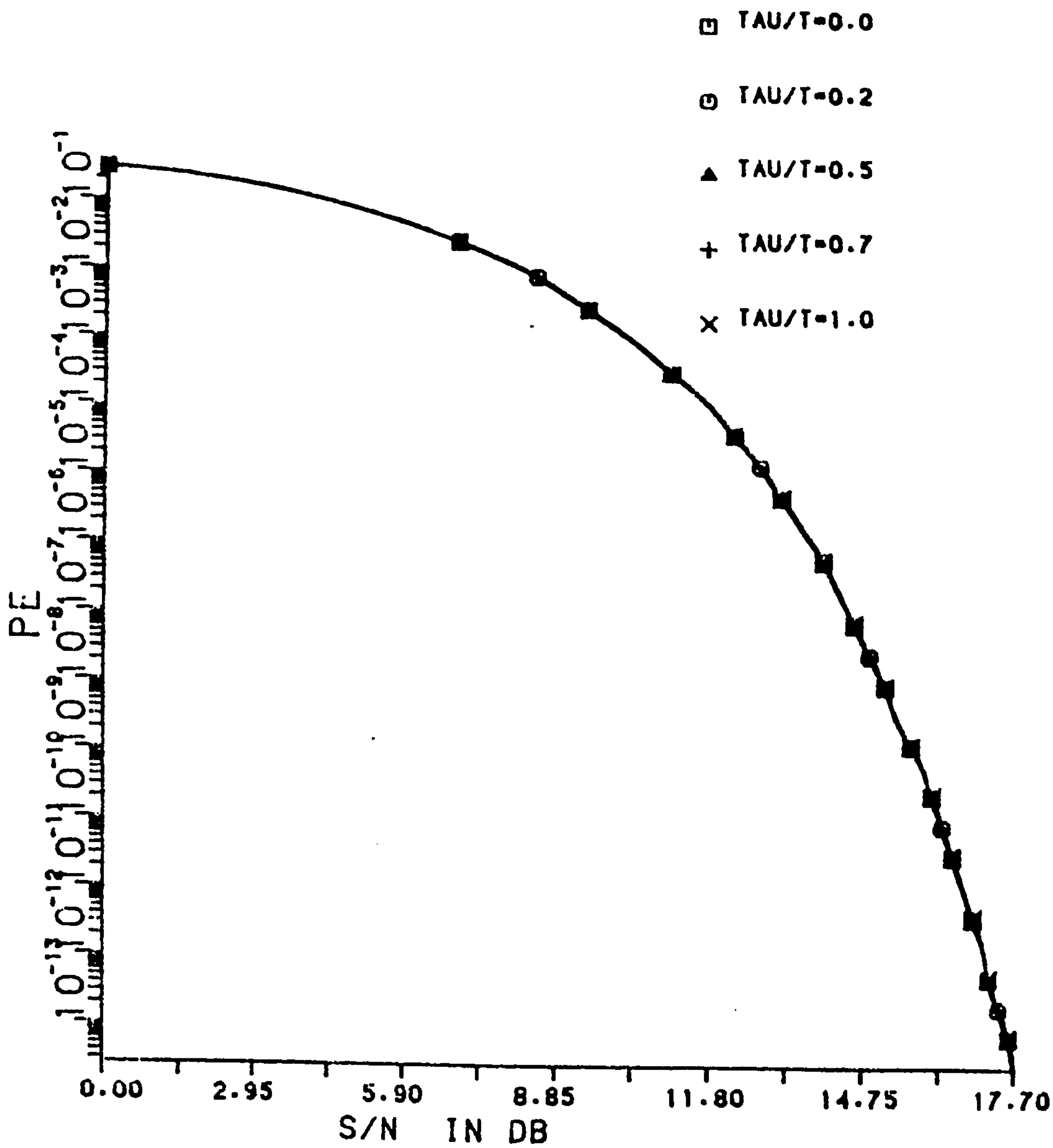


Fig 3.11 4-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (Delta = 0.0)

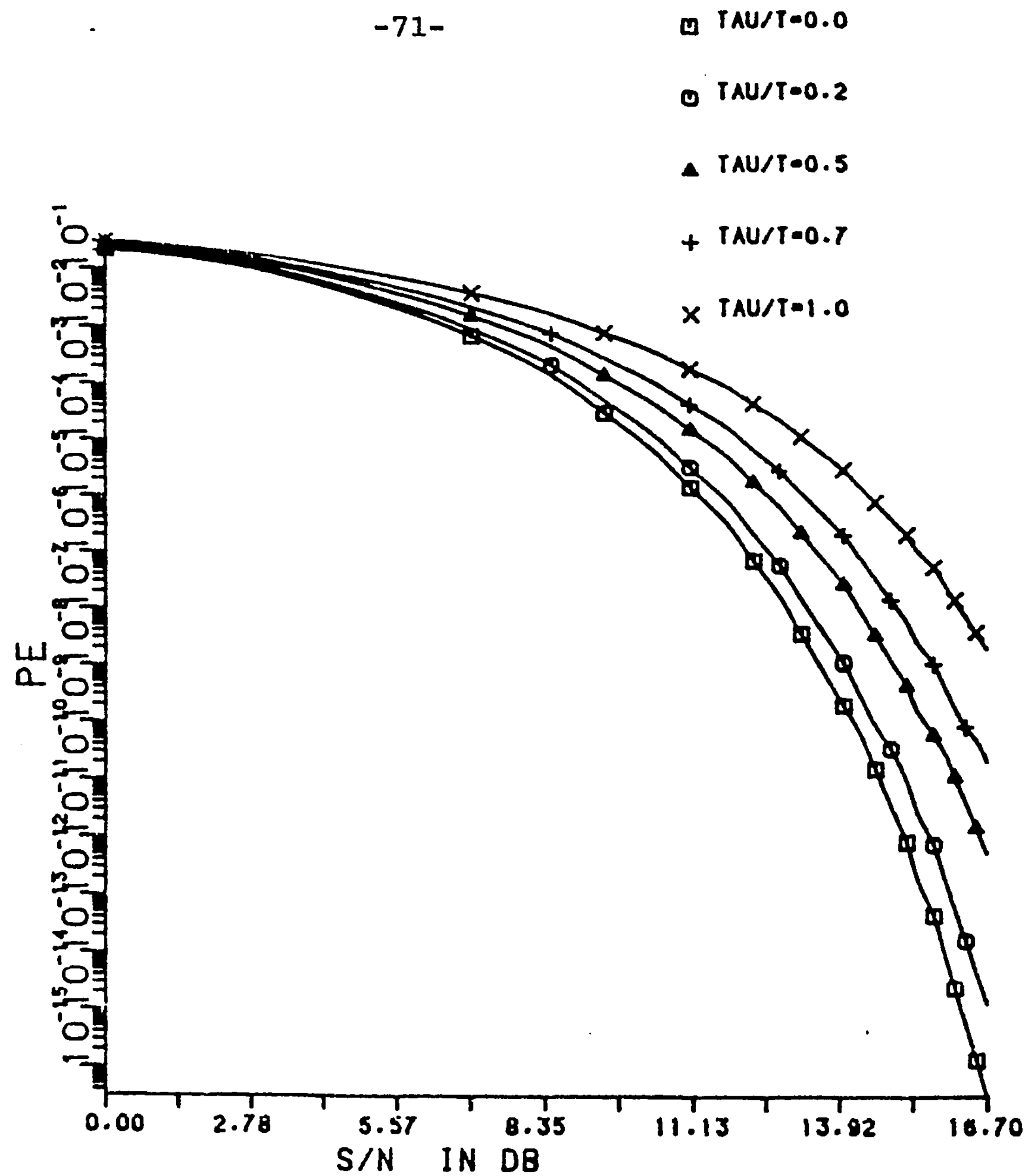


Fig 3.12 4-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (Delta = 0.2)

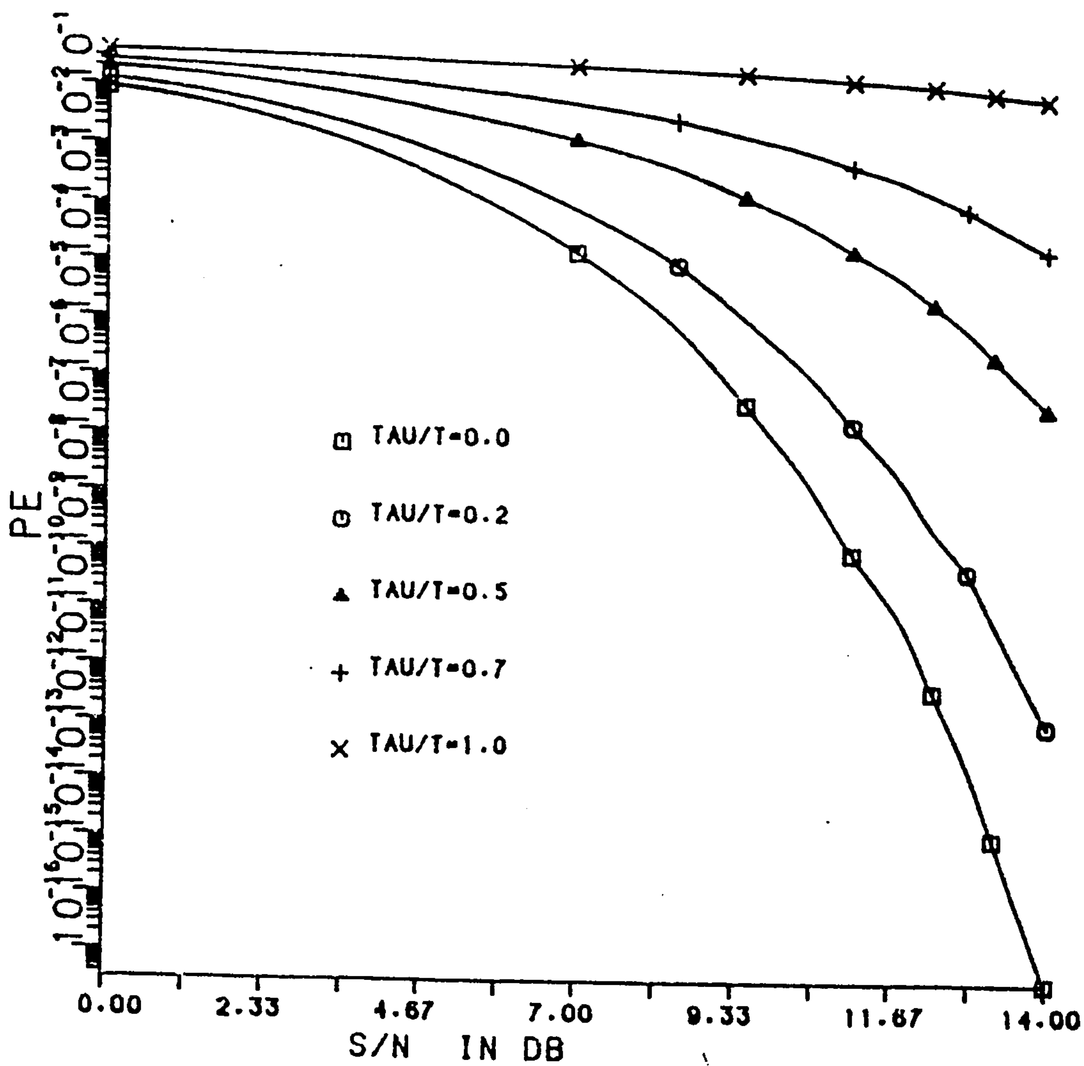


Fig 3.13 4-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (Delta = 0.7)

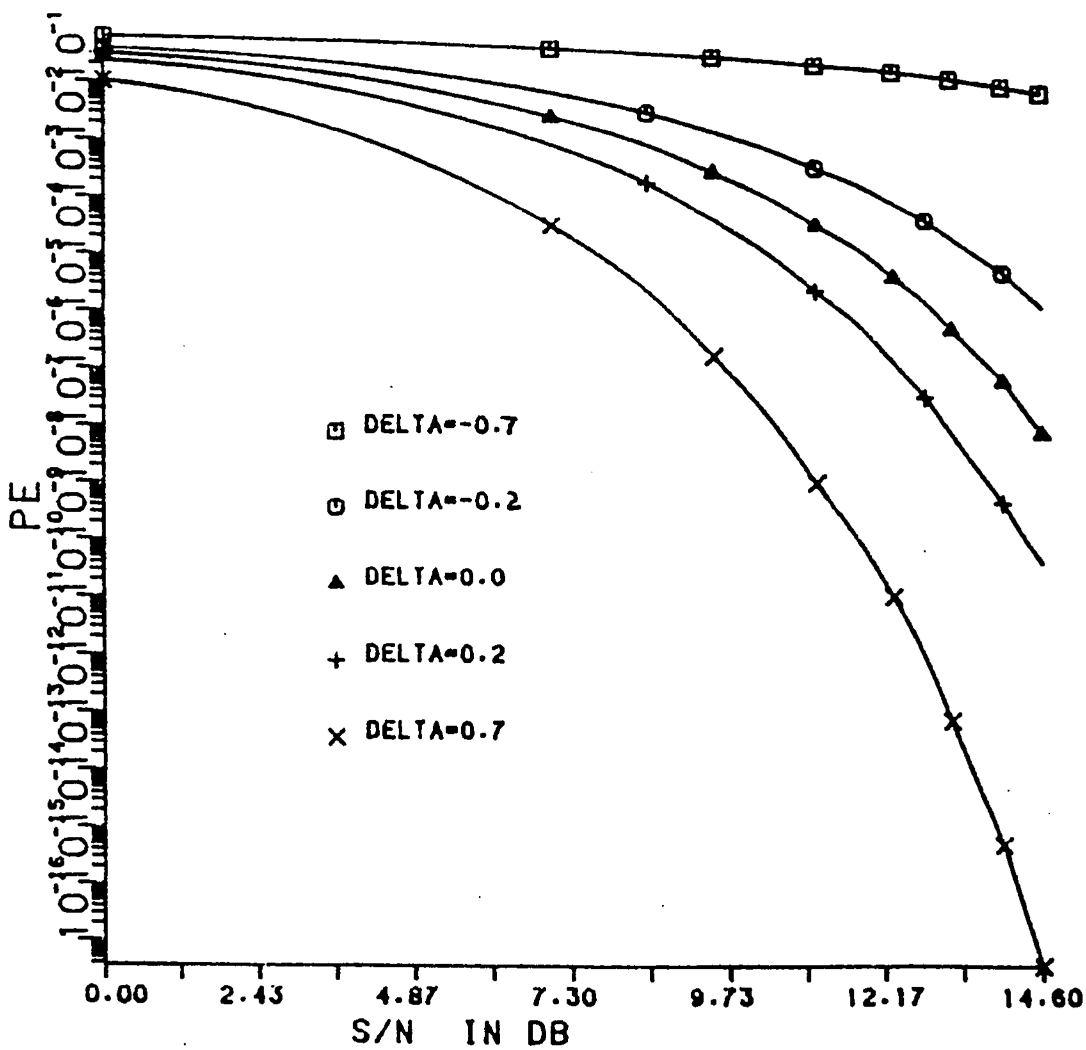


Fig 3.14 4-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (tau/T = 0.1)

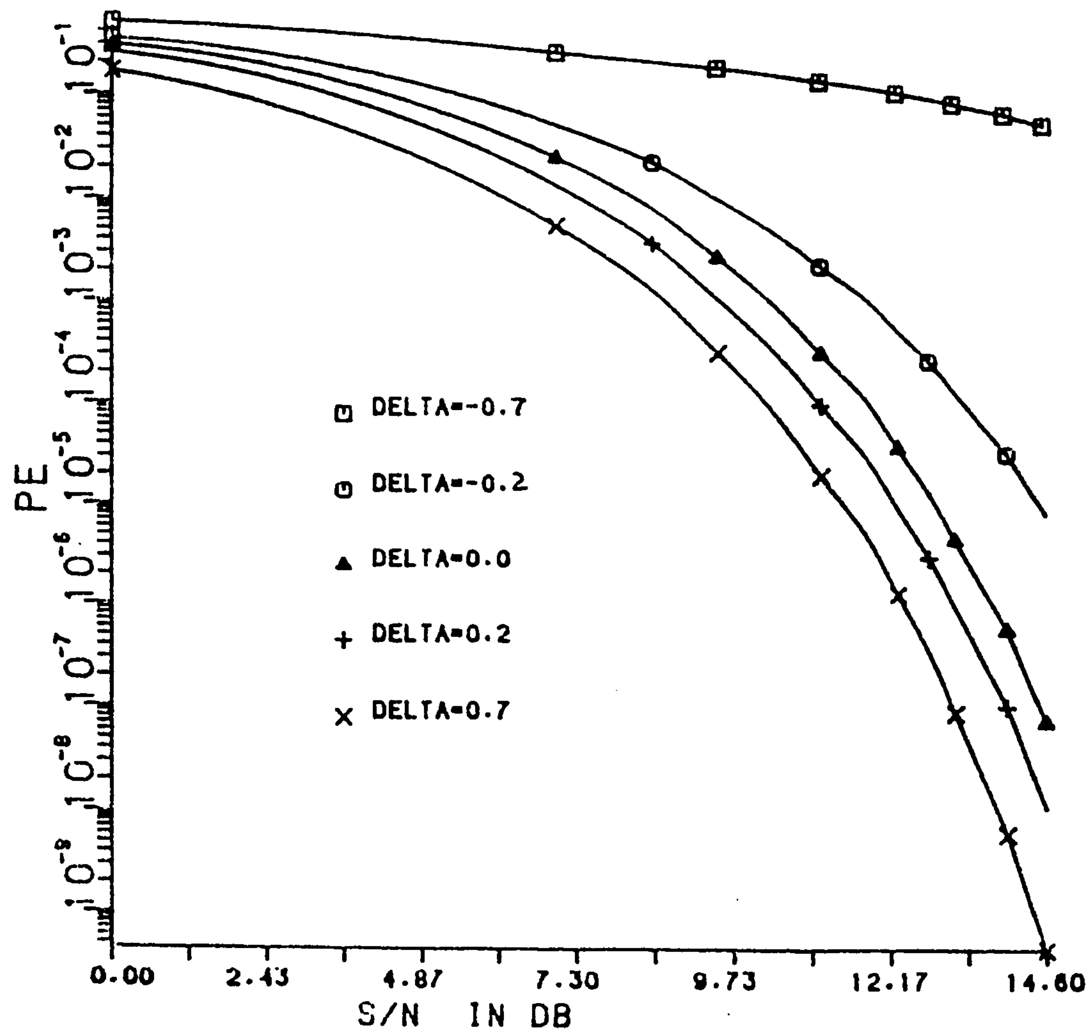


Fig 3.15 4-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (tau/T = 0.4)

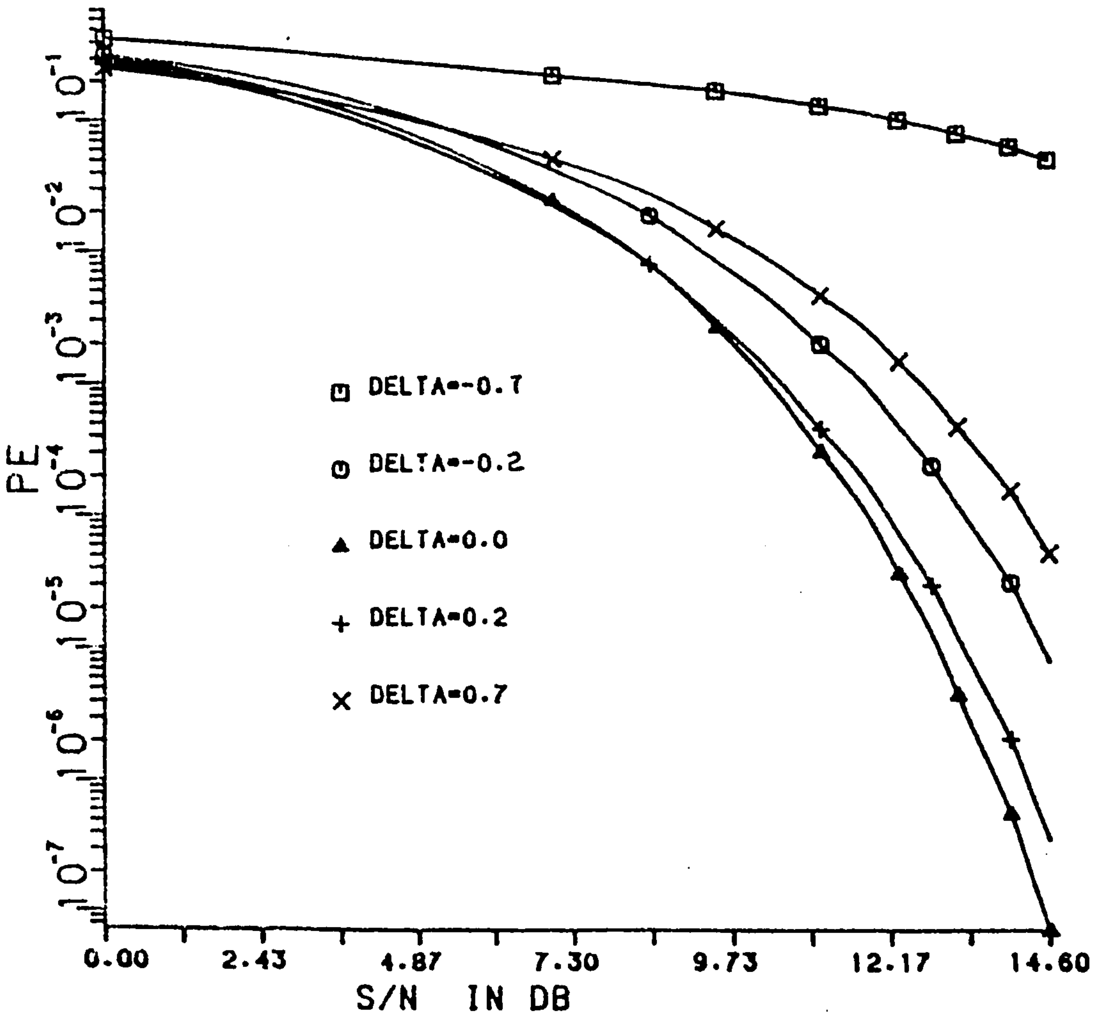


Fig 3.16 4-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (tau/T = 0.7)

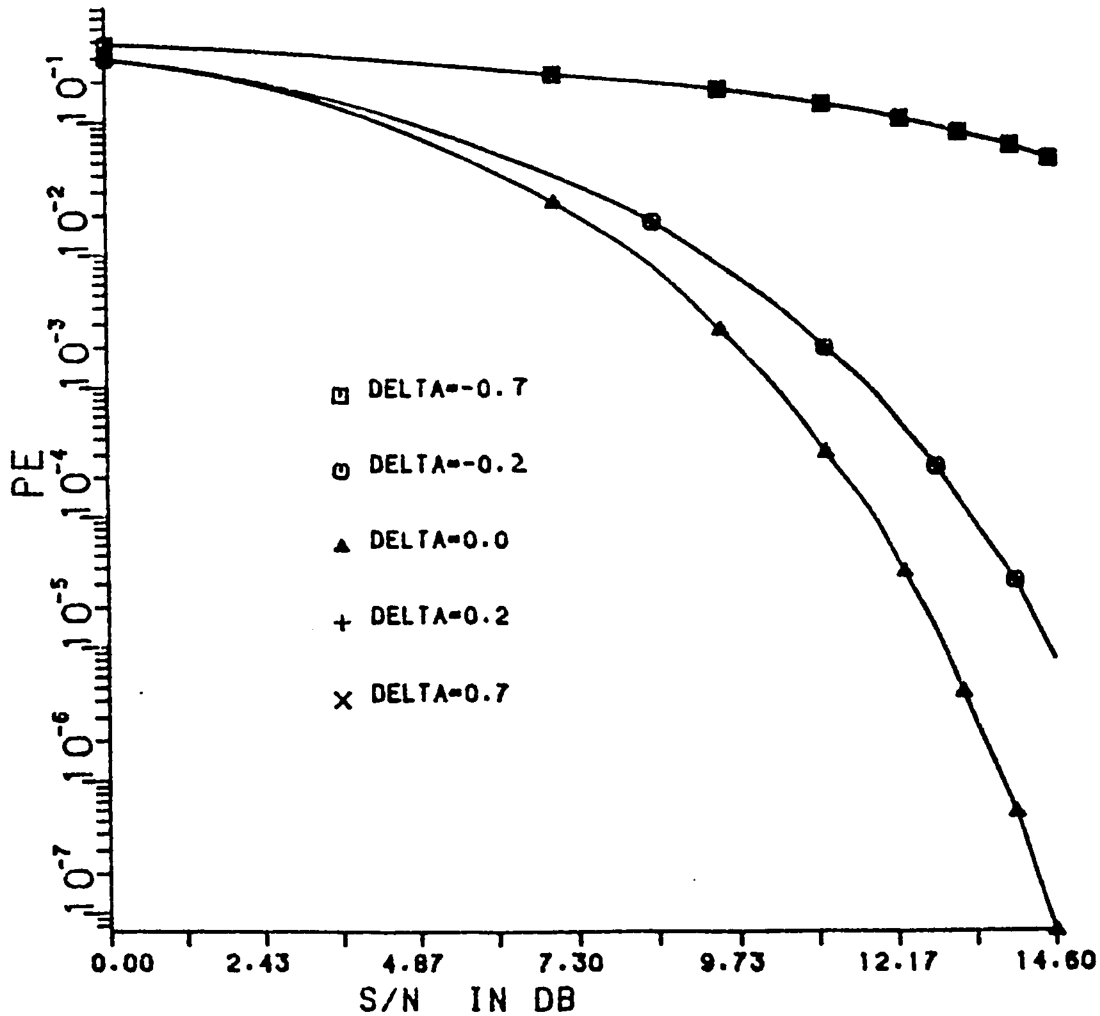


Fig 3.17 4-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (tau/T = 1.0)

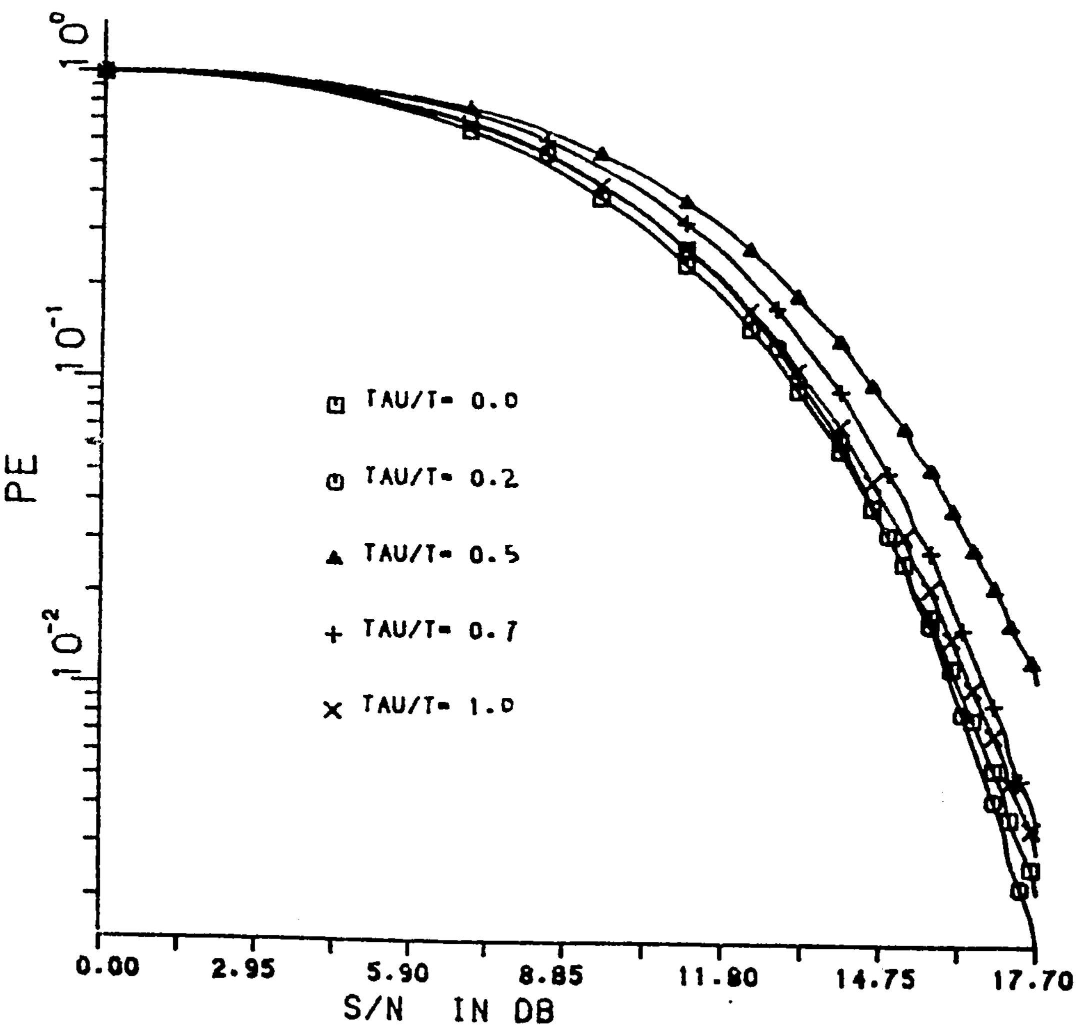


Fig 3.18 16-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (beta = 0.1)

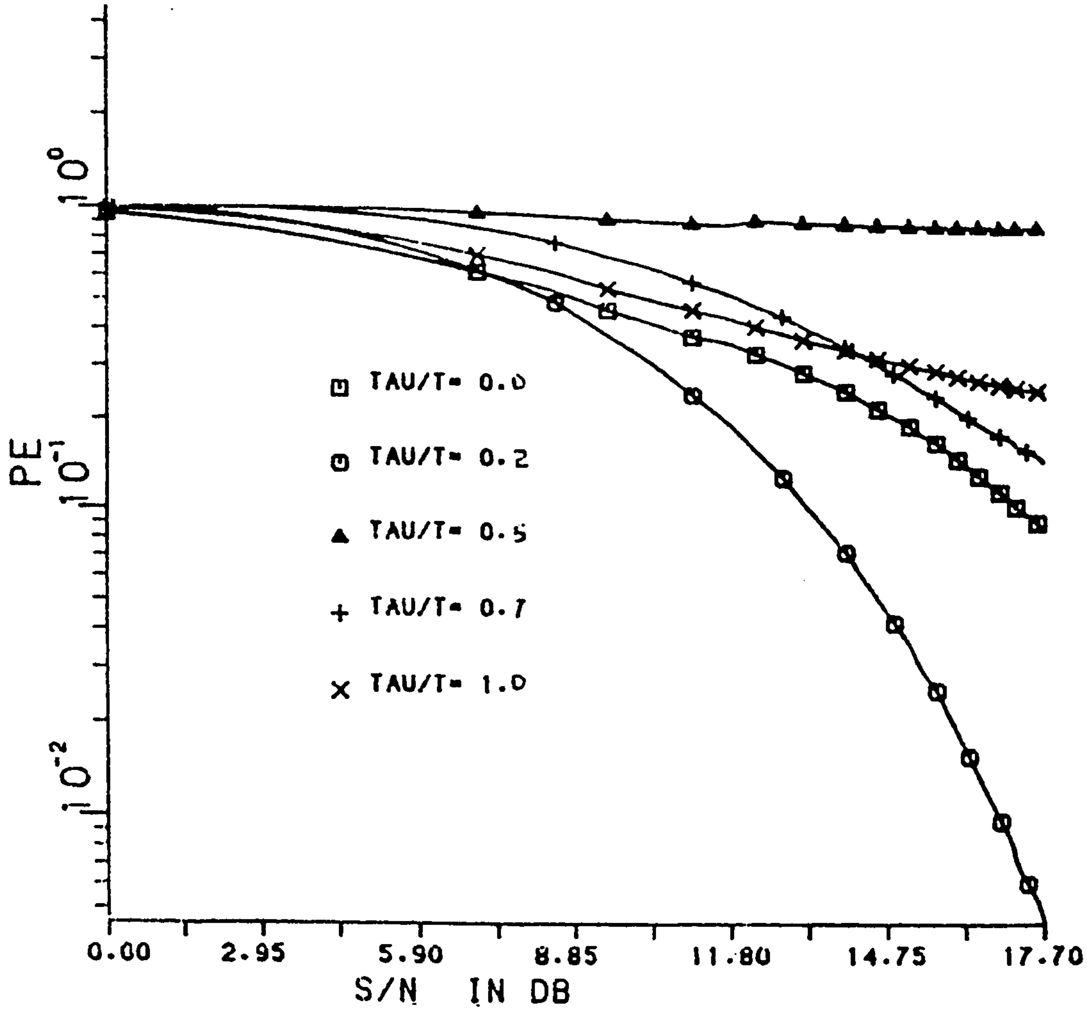


Fig 3.19 16-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (beta = 0.5)

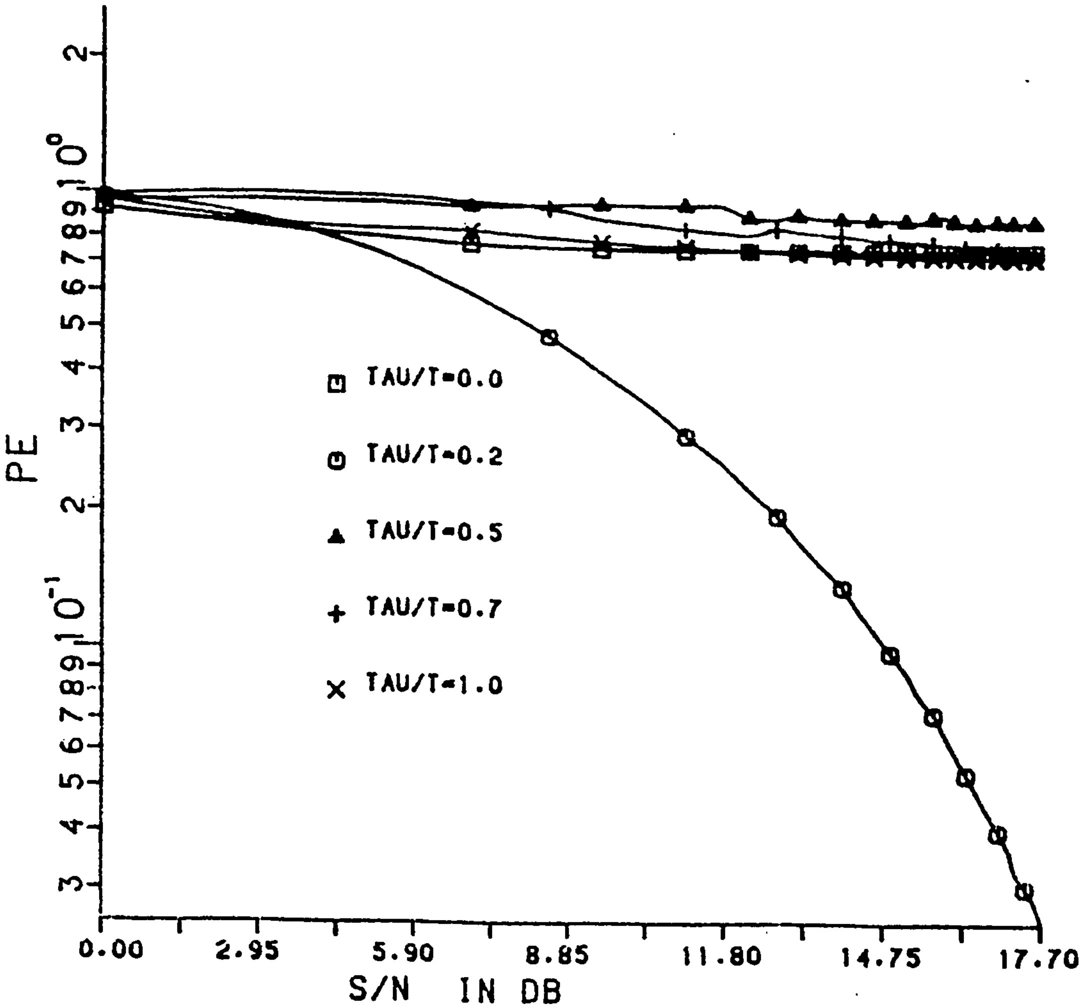


Fig 3.20 16-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (beta = 1.0)

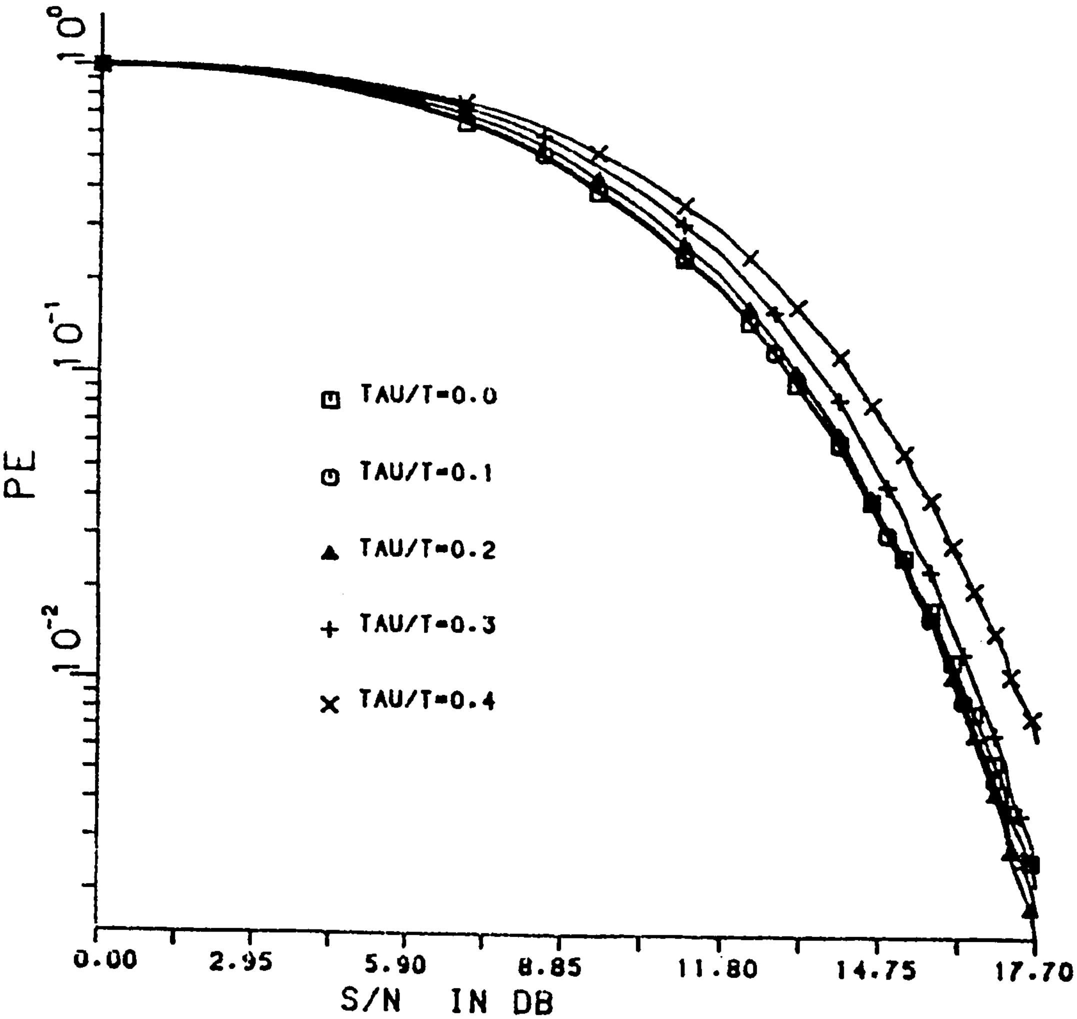


Fig 3.21 16-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (beta = 0.1 and 0.0 < tau/T < 0.4))

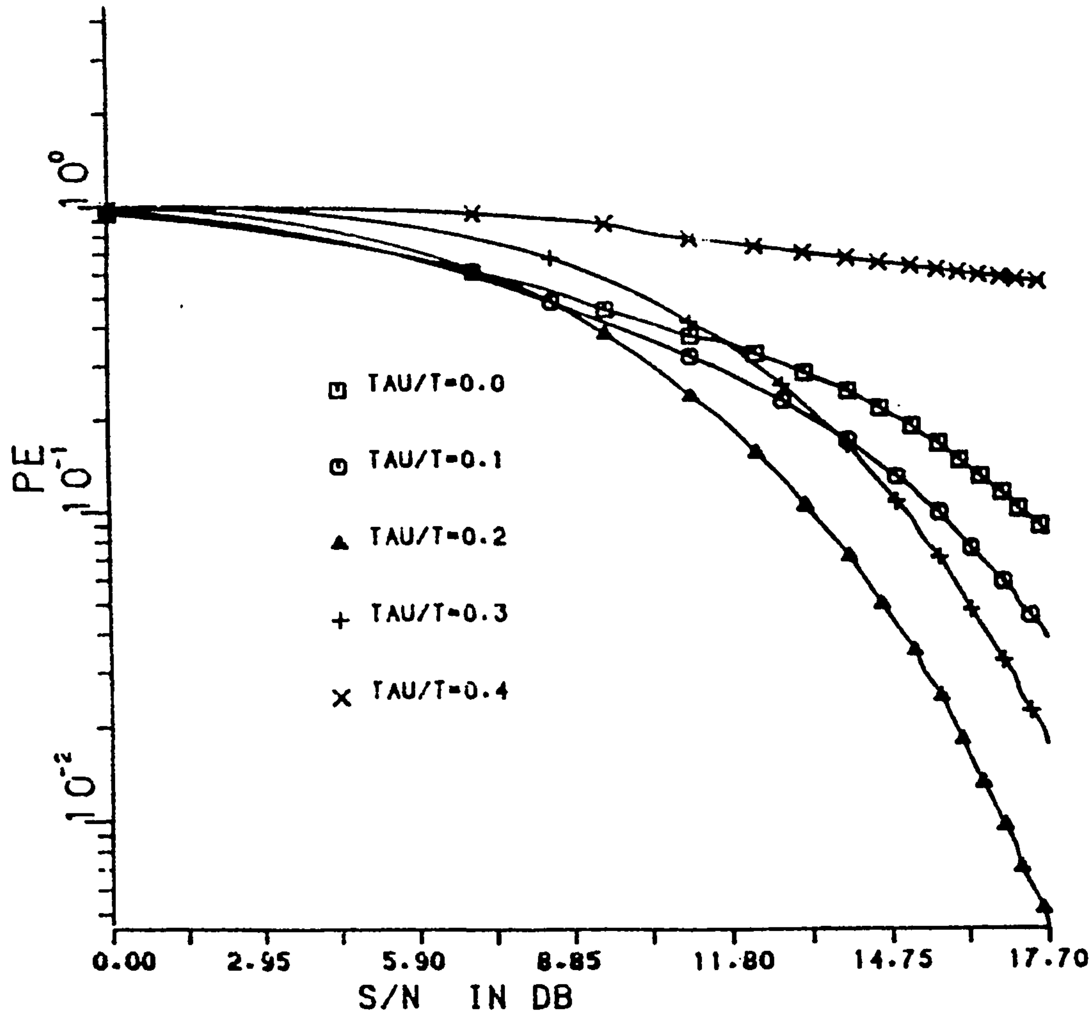


Fig 3.22 16-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (beta = 0.5 and 0.0 < tau/T < 0.4))

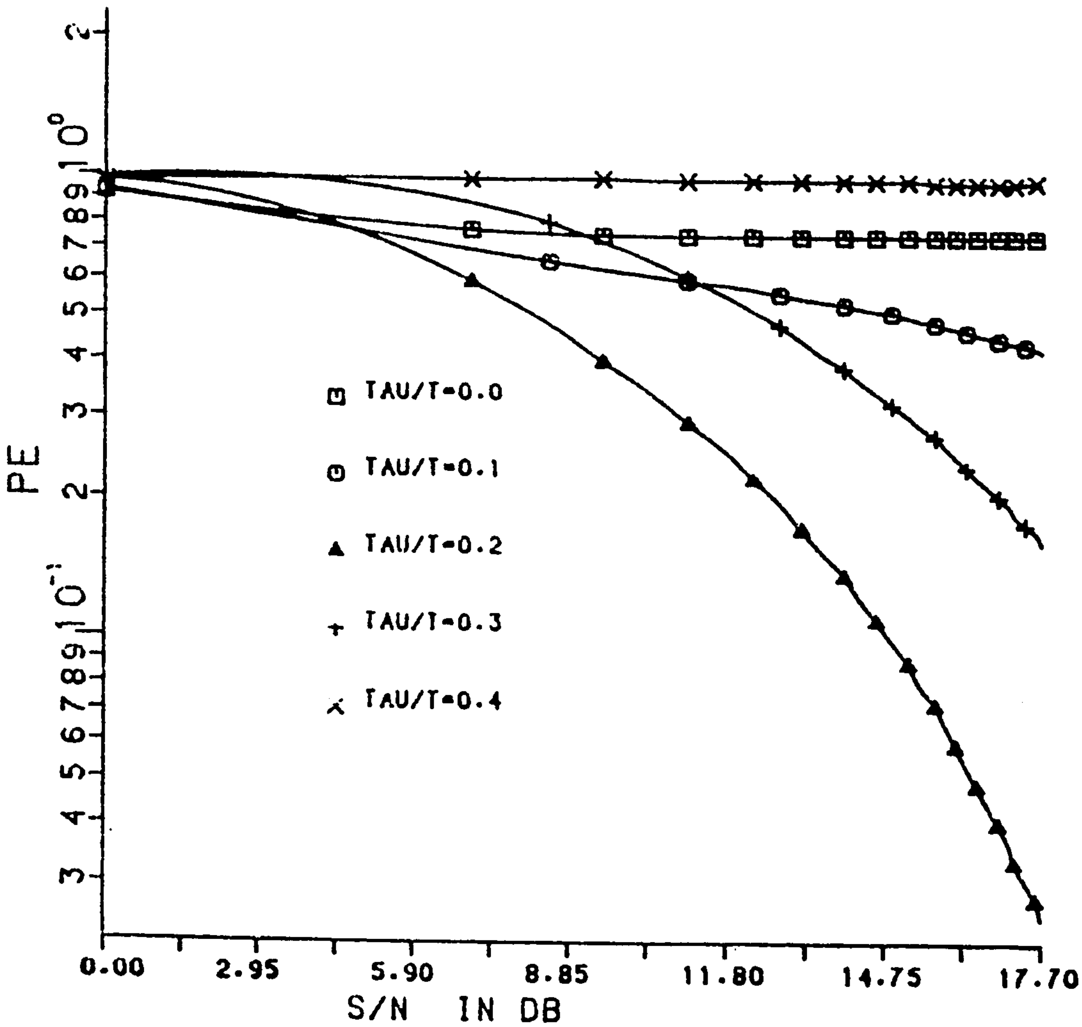


Fig 3.23 16-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (beta = 1.0 and 0.0 < tau/T < 0.4))

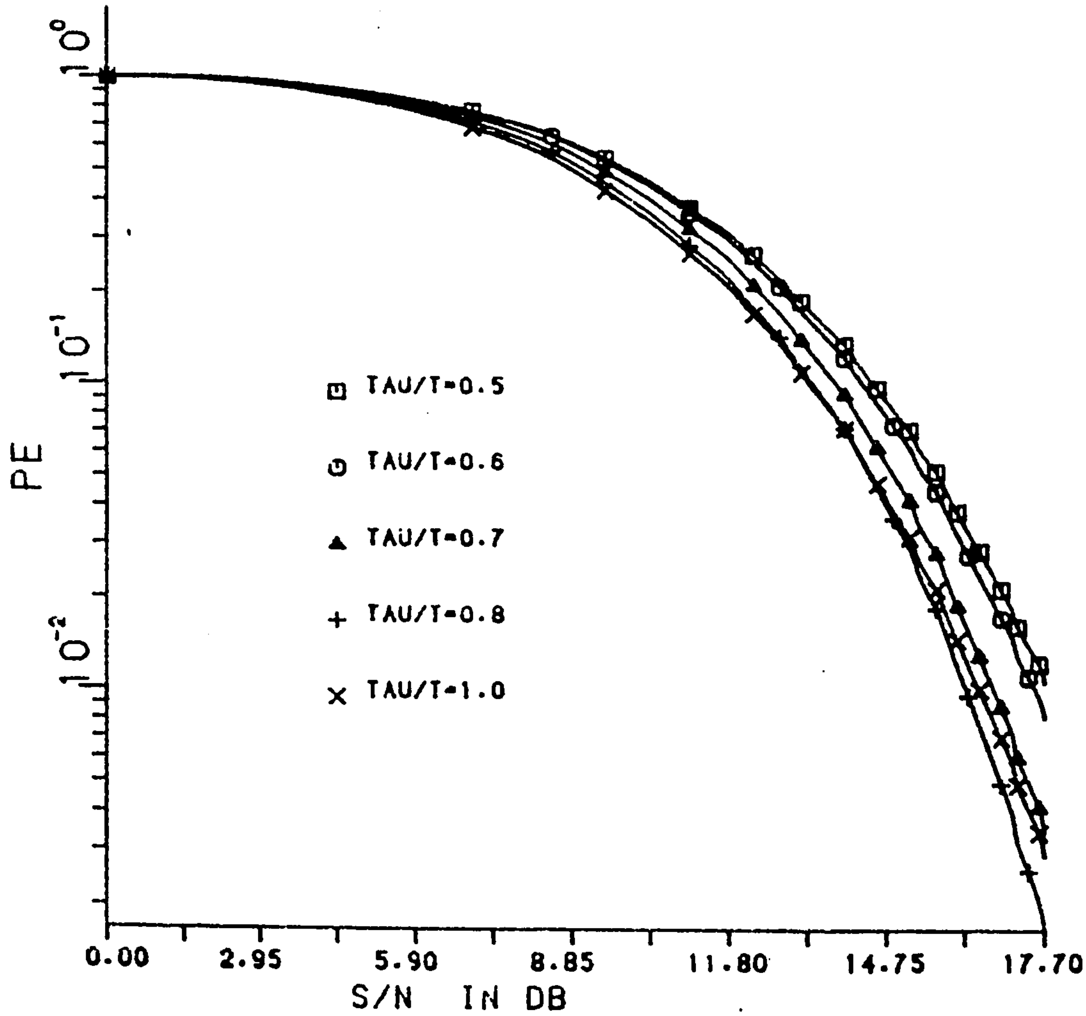


Fig 3.24 16-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (beta = 0.1 and 0.4 < tau/T < 1.0))

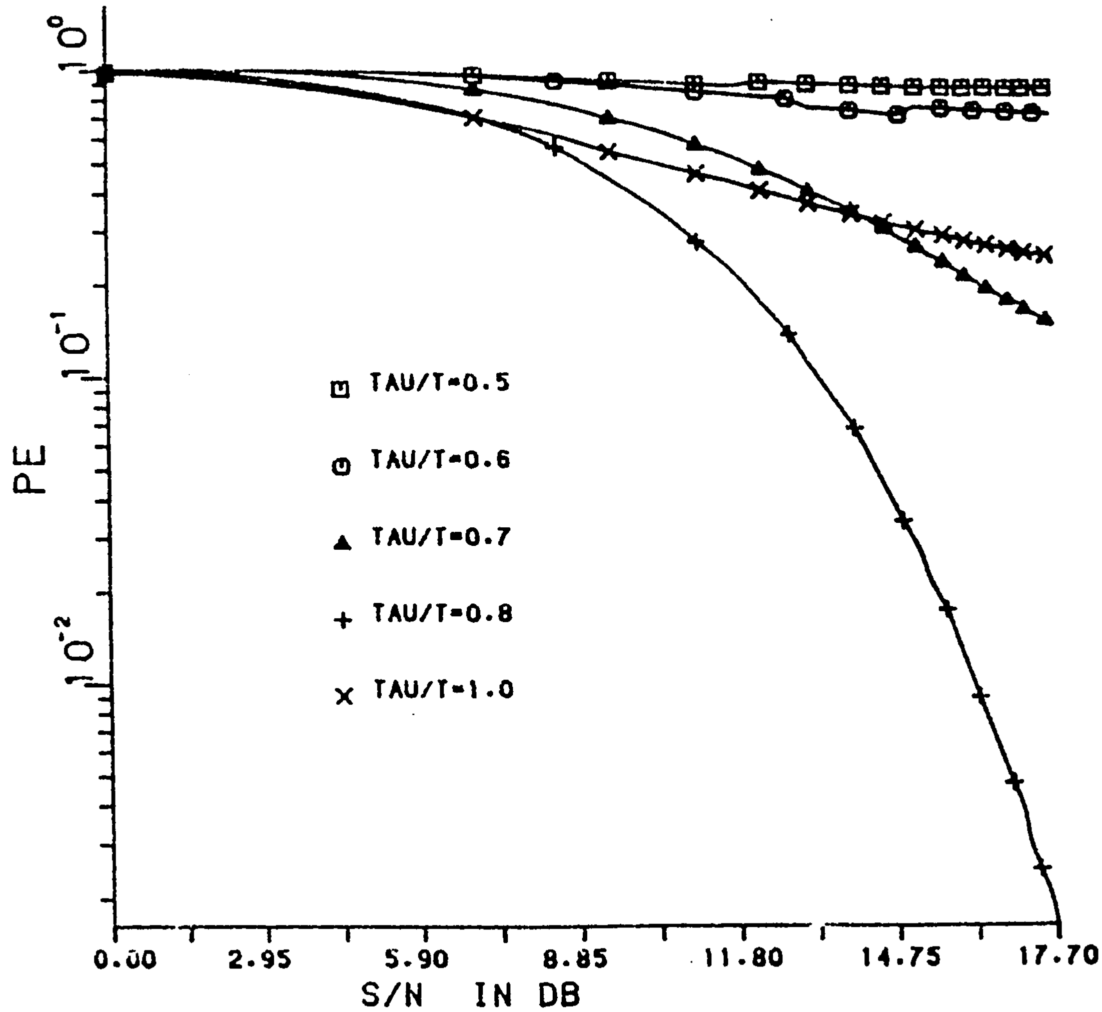


Fig 3.25 16-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (beta = 0.5 and 0.4 < tau/T < 1.0))

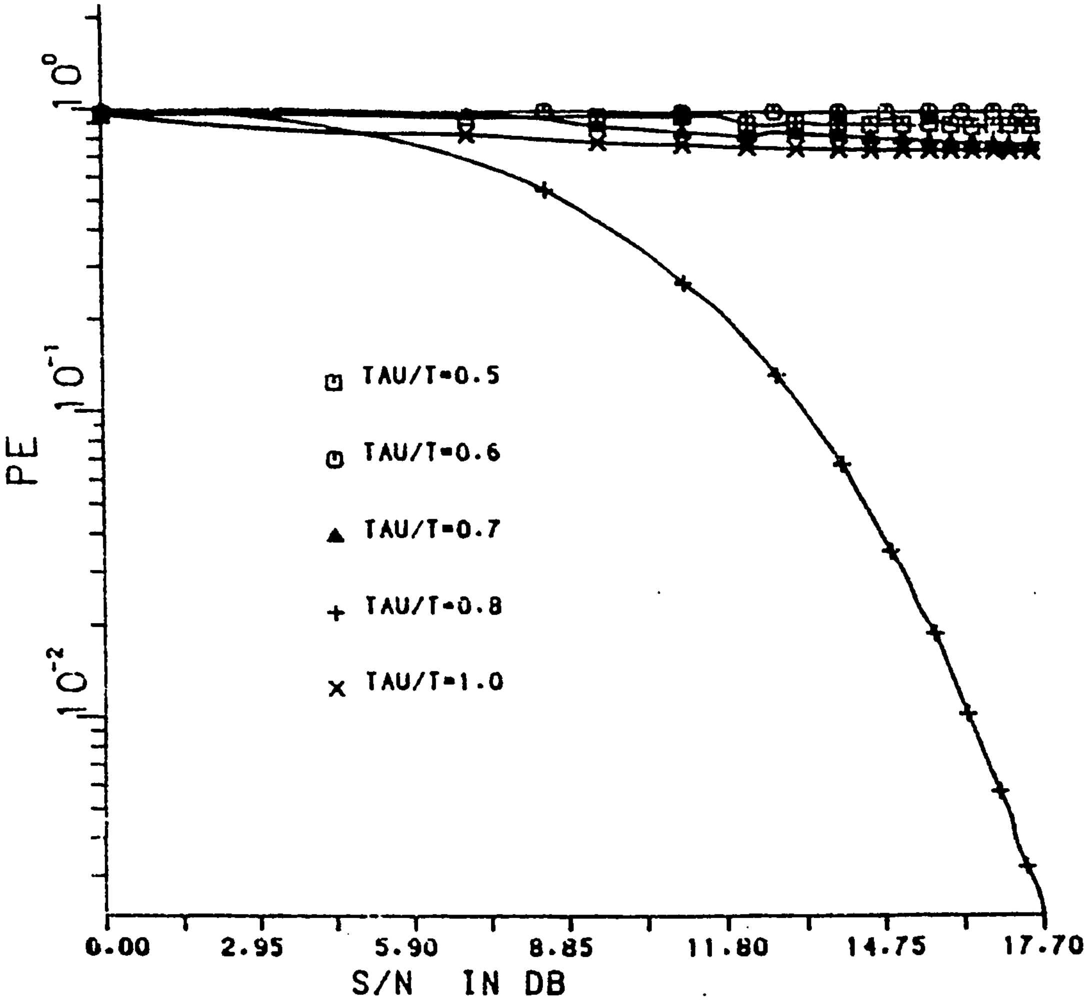


Fig 3.26 16-QAM Pe evaluation with S/N in presence of Frequency-Selective Fading (beta = 1.0 and 0.4 < tau/T < 1.0))

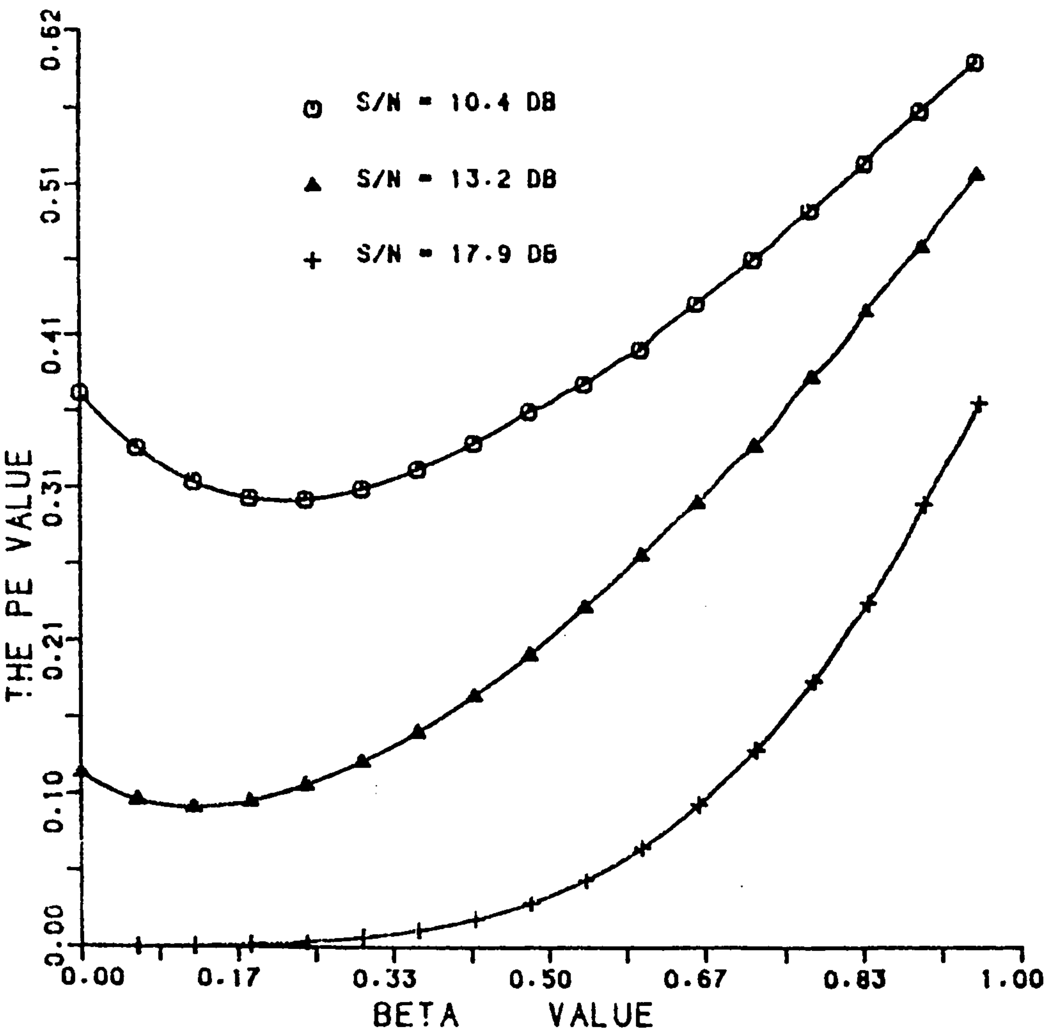


Fig 3.27 16-QAM Pe evaluation with S/N and beta in presence of Frequency-Selective Fading (tau/T = 0.1)

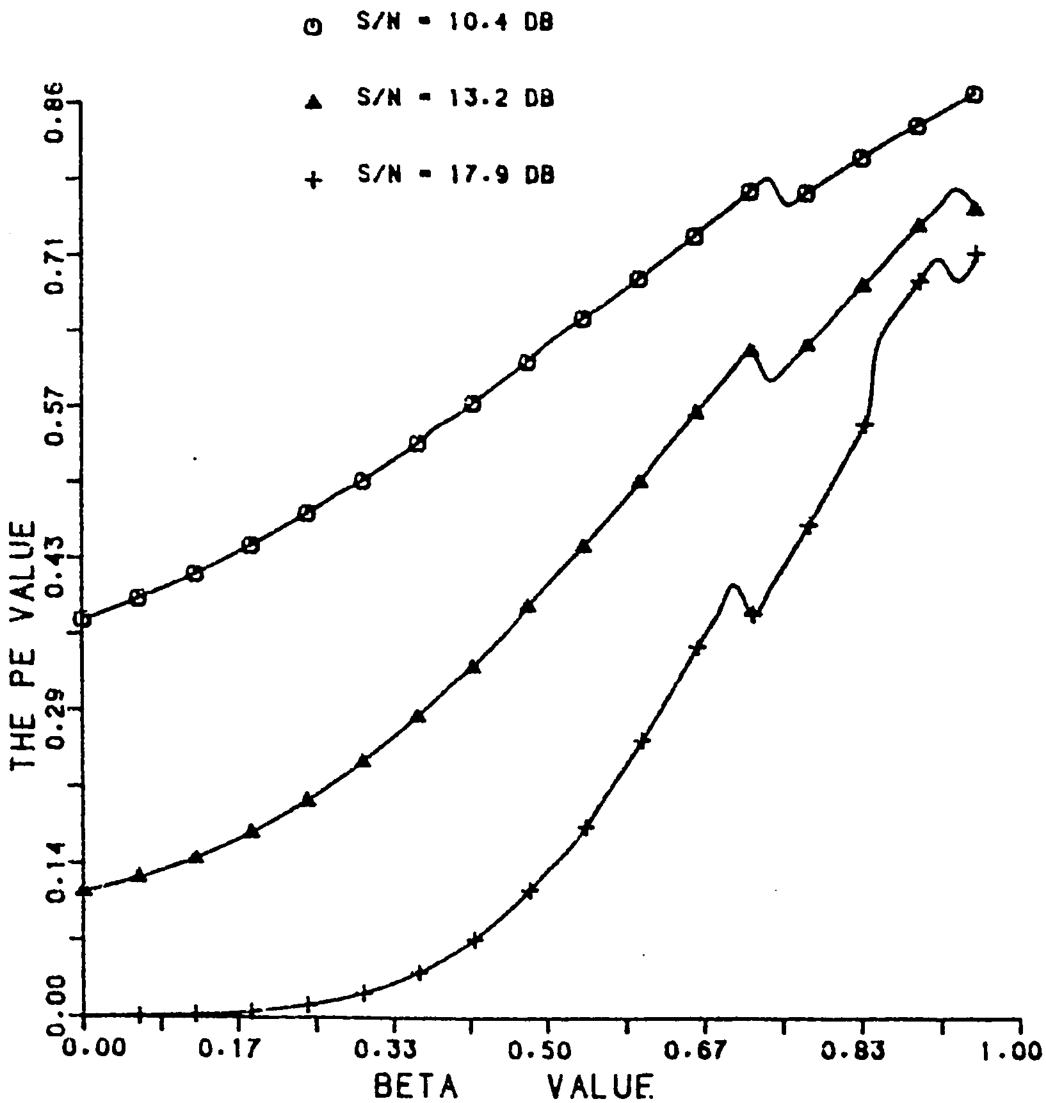


Fig 3.28 16-QAM Pe evaluation with S/N and beta in presence of Frequency-Selective Fading (tau/T = 0.7)

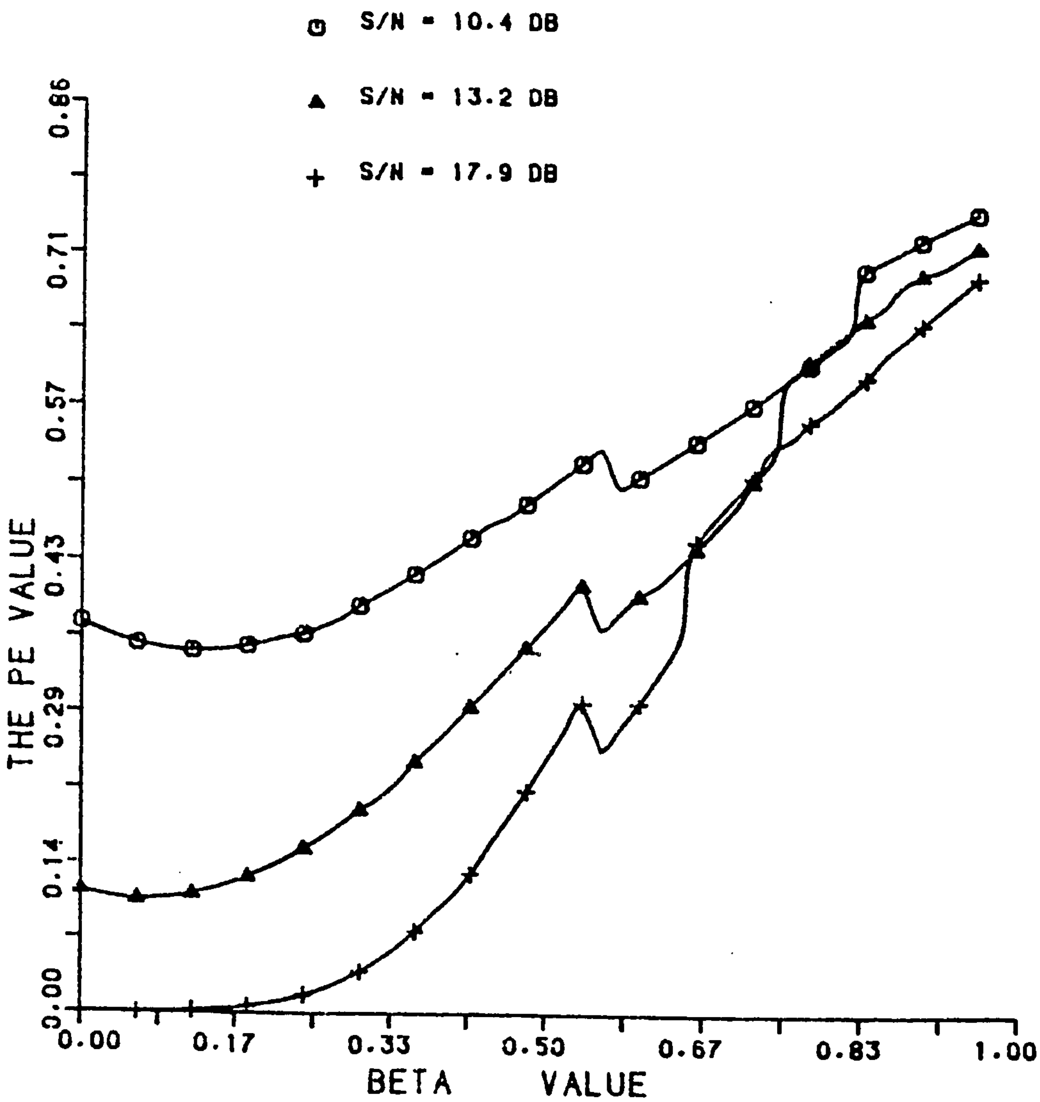


Fig 3.29 16-QAM Pe evaluation with S/N and beta in presence of Frequency-Selective Fading (tau/T = 1.0)

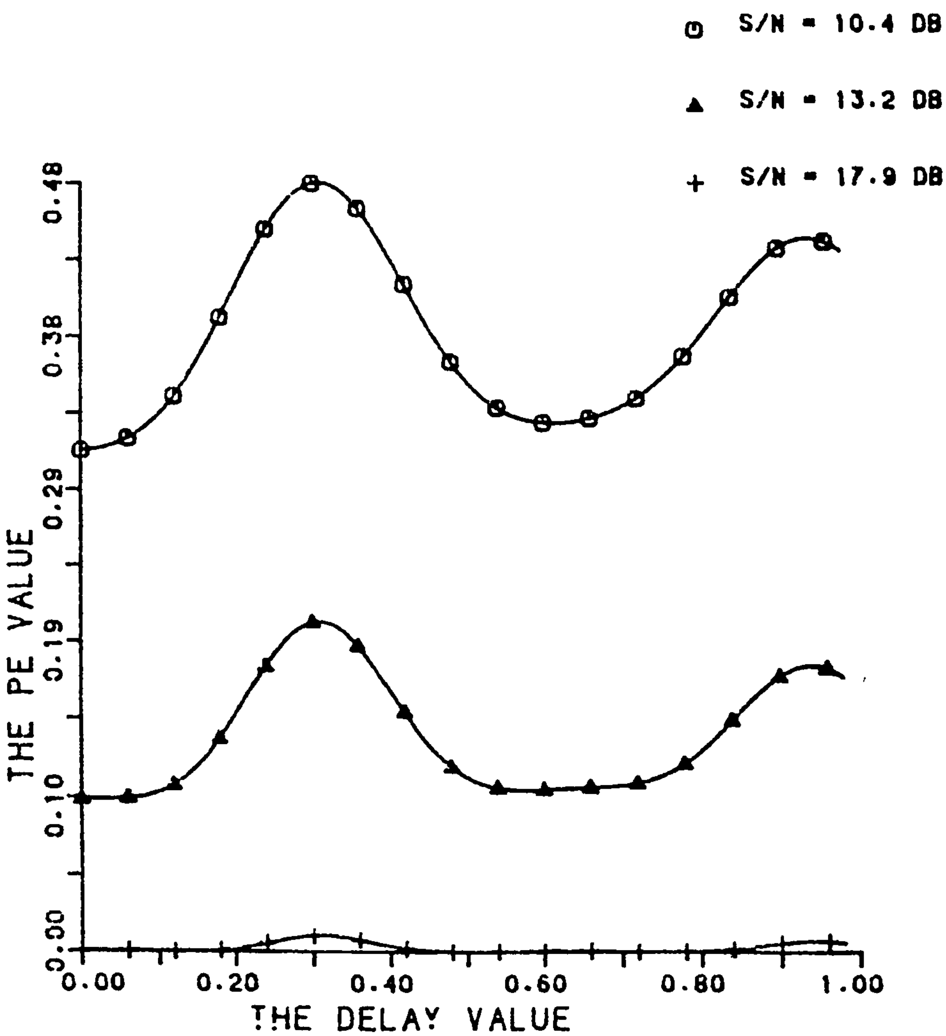


Fig 3.30 16-QAM Pe evaluation with S/N and tau/T in presence of Frequency-Selective Fading (beta = 0.1)

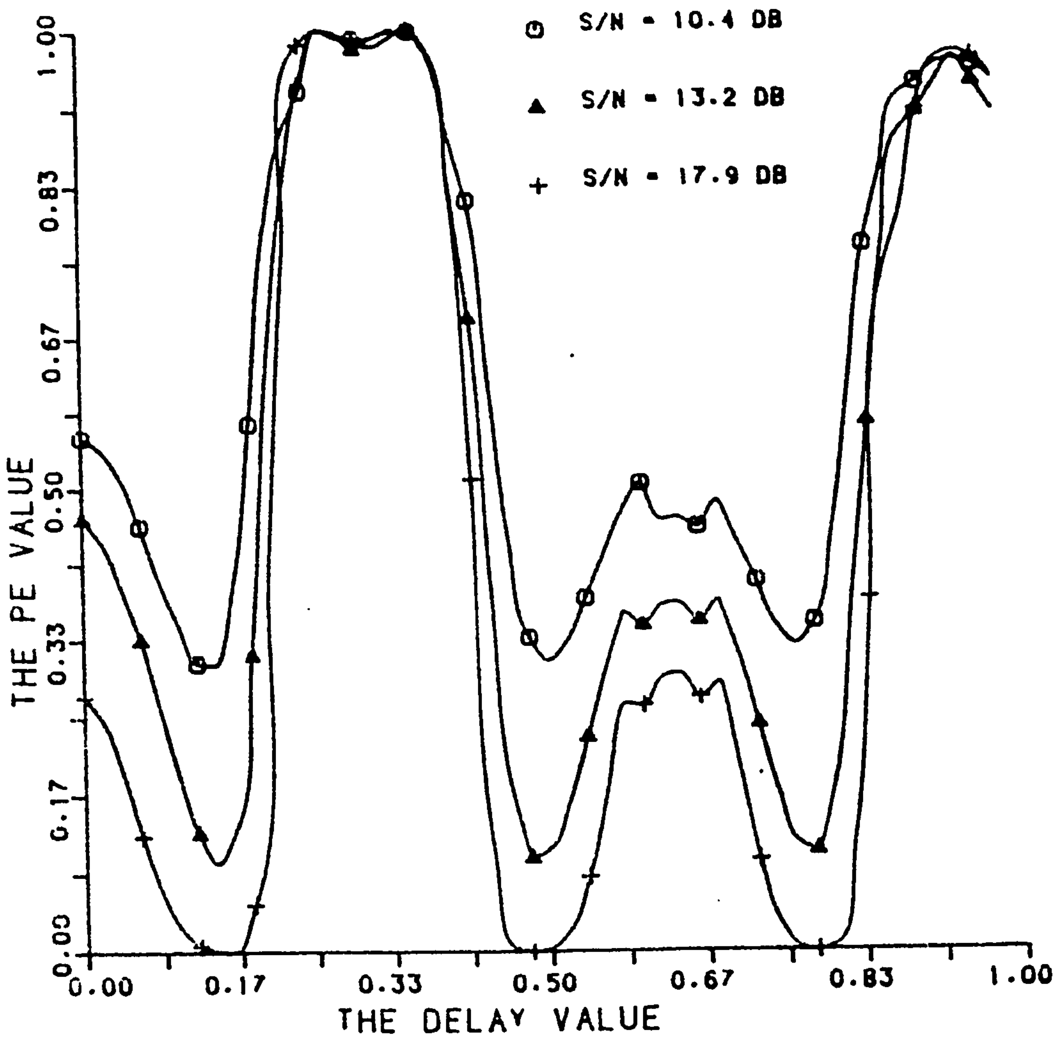


Fig 3.31 16-QAM Pe evaluation with S/N and tau/T in presence of Frequency-Selective Fading (beta = 0.7)

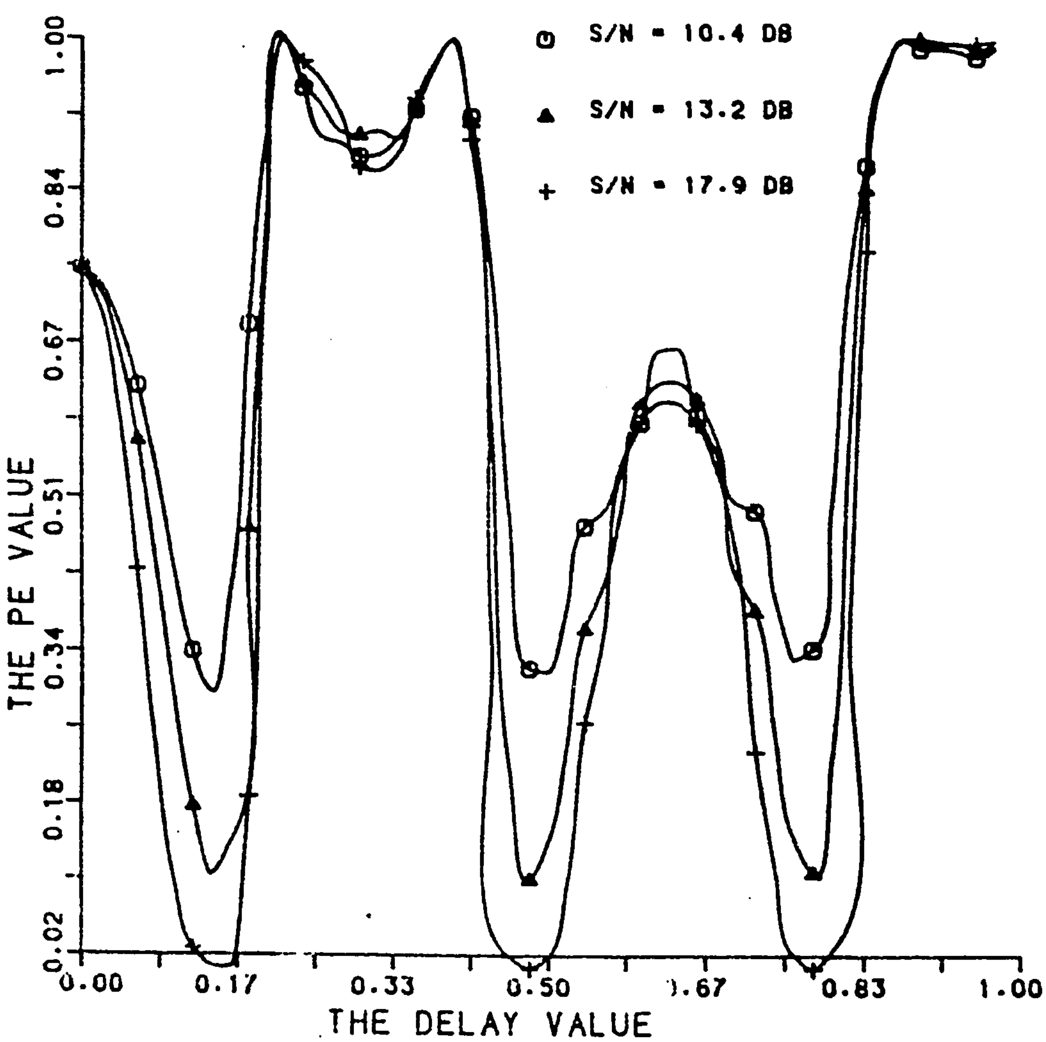


Fig 3.32 16-QAM Pe evaluation with S/N and tau/T in presence of Frequency-Selective Fading (beta = 1.0)

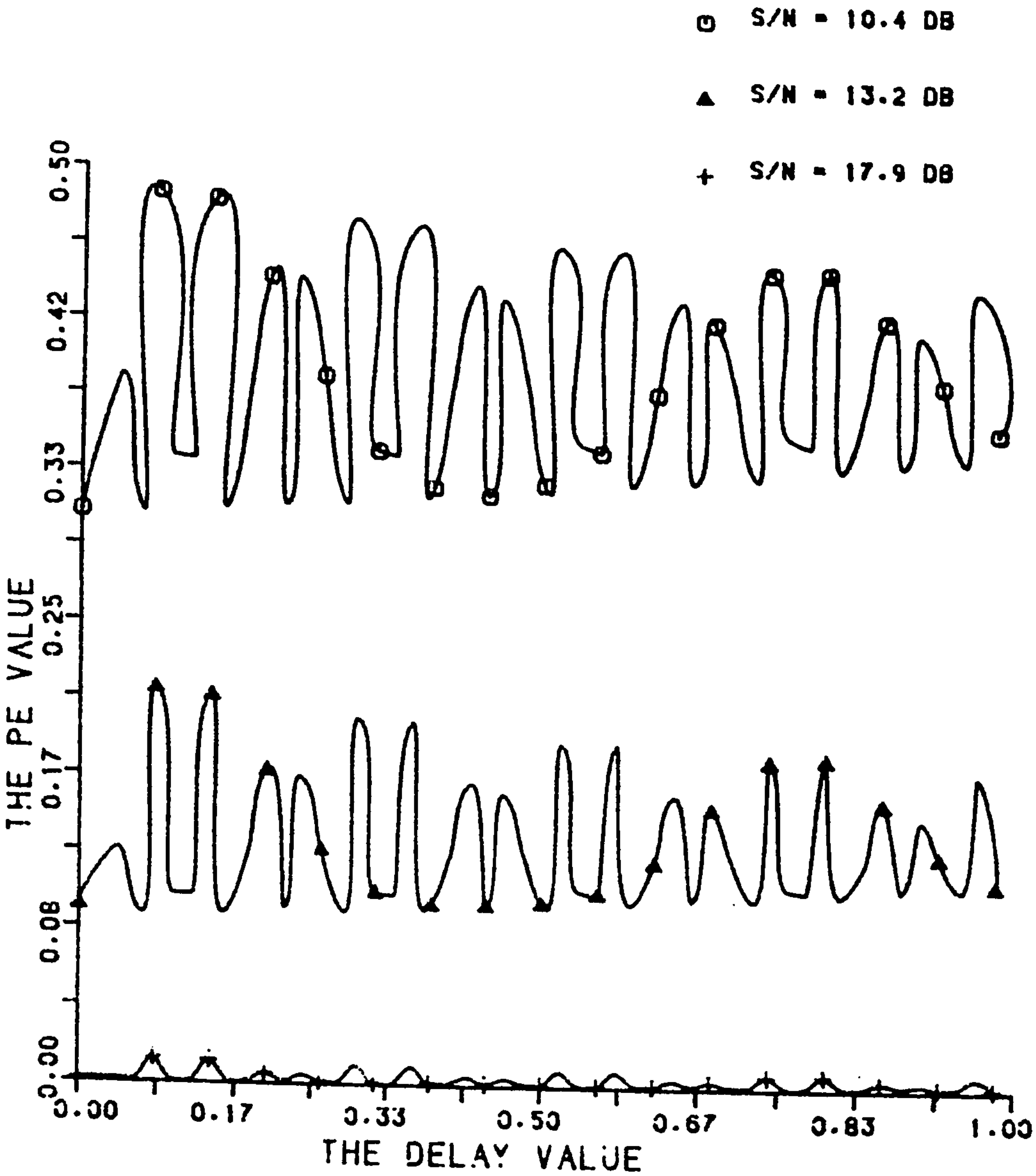


Fig 3.33 16-QAM Pe evaluation with S/N and tau/T in presence of Frequency-Selective Fading (beta = 0.1, and foT = 100)

CHAPTER FOUR

DIGITAL EQUALIZATION FOR MULTIPATH FADING

4.1: Introduction

The main protection system used so far to combat the dramatic effects of MPF, which manifests through ISI generation, is the equalizers. They are employed either at the IF or the BB system sections, but for achieving high degree of distortion cancellation, an IF and BB equalizer arragments are used in the communication system. The conventional equalizers were of the zero-forcing type, but due to the random characteristics of the MPF event, a transmitted testing sequence should be all time used to adapt the equalizer settings and to accomodate the continious channel variations. Hence, An adaptive equalizer becomes necessary.

Our work in this chapter deals with a datailed analysis on the two types of the equalizers using the different models and investigate the impact of the channel parameters on the performance of the equalizers. The theory is reported in [15]. The analysis done in this chapter is performed on the BB section of the radio link to investigate the performance of the equalizers with the different channel models.

4.2: Zero Forcing Equilizer

Let write the impulse response in a sampled vector form

$$H = h_o\ h_1\ h_2\ \ldots\ldots\ldots\ h_g \qquad (4.1)$$

where, $\qquad h_i = h(iT)$

Using Z transform, the sampled impulse response is given by

$$H(z) = h_o + h_1z^1 + h_2z^{-2}\ \ldots\ldots\ + h_gz^{-g} \qquad (4.2)$$

where z^{-1} represents the time instant t=iT.

For the ith transmitted signal element, s_iz^{-i}, the z transform of the ith received signal at the sampler output would be:

$$s_iz^{-i}H(z) = s_ih_oz^{-i} + s_ih_1z^{-i-1} + \ldots\ldots + s_ih_gz^{-i-g} \qquad (4.3)$$

The task of the equalizer is to remove signal distortion or attenuation , so the optimum of the transmitted ith signal component would be

$$s_iz^{-i}H(z)C(z) \simeq s_iz^{-i-h}$$

where C(z) is the equalizer z transform representation and h is the hT delay introduced in the equalization process, and the s_i component is detected from the sample value x_{i+h} at the output of the equalizer at the time $t = (i+k)T$ from :

$$x_{i+k} \simeq s_i + u_{i+h} \tag{4.5}$$

where u_{i+h} is the gaussian noise component .

hence, $$C(z) = z^{-h} H^{-1}(z) \tag{4.6}$$

which means that the equalizer is the inverse of the channel with a delay of hT. The desired output is

$$CY = E_h$$

where, $$E_h = 0\ 0\ \ldots.1\ldots..0\ 0$$

but in reality, the expression would be

$$CY = E = e_o\ e_1\ \ldots\ldots..\ e_{m+g-1}$$

The peak distortion in the equalized signal is defined by

$$D_p = \frac{1}{|e_h|} \sum_{\substack{i=0 \\ i \neq h}}^{g} |e_i| \tag{4.7}$$

The mean-square distortion is

$$D_m = \frac{1}{e^2{}_h} \sum_{\substack{i=0 \\ i \neq h}}^{g} e^2{}_i \qquad (4.8)$$

and the mean-square error due to ISI is

$$D_i = k^2 \; | \; E - E_h \; |^2 \qquad (4.9)$$

For $k = \pm 1$ in the data stream,

$$D_i = | \; E - E_h \; |^{\#2}.$$

The equalized output is

$$CY = e_o \; \ldots \; e_{j+1} \; .. \; 0 \; 0 \; \ldots \; 1 \; \ldots \; 0 \; 0 \; \ldots\ldots \; e_{m+j-1}$$

Dp is minimized when $e_i = 0$ for $h-1 < i < h$ and $e_h = 1$

The complete results are reported in the subsequent sections.

4.3: *Minimum mean-square error equalizer*

The detected output due to the transmitted component s_i

$$x_{i+h} = \sum_{j=0}^{m} s_{i+h-j} \, e_j + u_{i+h}$$

Ideally, the received signal would be

$$x_{i+h} = s_i + u_{i+h}$$

The linear equalizer which minimizes the mean-square error in the output signal, minimizes the mean-square value of

$$\zeta = E\{(x_{i+h} - s_i)^2\}$$

$$= k^2 \; (e_{h-1}) + \sum_{j=0}^{m} k^2 \; e_j^2 + E[u^2{}_{i+h}]$$

$$= k^2 \; | \; E-E_h \; |^2 + \sigma^2 \; |C|^2 \qquad (4\text{-}10)$$

where σ^2 in the gaussian noise variance.

The term $k^2 \; |E-E_h|^2$ is the MSE in x_{i+h} due to ISI and σ^2 is the MSE due to Gaussian noise.

but $\qquad$ CY = E, so

$$\zeta = k^2 \; | \; CY-E_h \; |^2 + \sigma^2 \; |C|^2$$

$$= C \; (\; k^2 \; YY^T + \sigma^2 \; I \;) \; C^T - 2k^2 \; E_h \; Y^T \; C^T + k^2$$

where I is an M x M identity matrix .

Following the derivation given in [15], the quantity ζ is minimized when

$$| \; CG - k^2 \; E_h \; Y^T \; (\; G^T \;)^{-1} \; | = 0$$

where G is defined as

$$GG^T = k^2 \ YY^T + \sigma^2 \ I$$

finally

$$C_{opt} = k^2 \ Y^T \ \{ \ YY^T + \{ (\frac{\sigma^2}{k^2}) \ I \ \}^{-1}$$

at high S/N ratio, $k^2 \;\; >> \;\; \sigma^2$

$$C_{opt} = k^2 \ Y^T \ \{ \ YY^T \ \}^{-1} \tag{4.11}$$

4.4: Results Analysis

The complete analysis is done with a computer program reported in APP-IV The strategy of the analysis is as following:

-1:The channel transfer function is transformed by Inverse Fast Fourier Transform to get a vector of the transfer function samples in time domain. the transformation gives an easy way to manipulate the model parameters.

-2:A data stream vector is multiplied by the channel vector.

-3:The resulting data stream vector is processed by the Zero Forcing equalizer and then by the Minimum mean square equalizer

-4:The tap coefficients and peak distortion , mean square distortion, and mean square error or ISI are evaluated and analysed.

The Two-Ray model has been empolyed as a basis for the analysis with the Z.F and MSE equalizers, this is illustrated in Figs.(4.1-3). The Three-Ray Model in Figs.(4.4-6), and similarly for the Polynomial model in Figs.(4.7-8).

With the Two-ray model, the Z.F and RMS equalizers succeed in minimizing Dp and Dm only for a small delay or a delay near bit duration value as seen in Figs.(4.1-2)and

Figs.(4.4-5), however, the MSE equalizer removes ISI better than the Z.F one, this is shown in Fig(4.3) and Fig(4.6). The latter Figure reveals small fluctuations of ISI aroud a mean value, in fact the other parameters Dp and Dm have the same behaviour, but this is not clear in the corresponding Figs.(4.1-5).

For the polynomial model, the equalizer also succeeds to remove the different types of distortion, as depicted by Figs.(4.7-8). However, the main disadvantage lies in the weakness of the model to be a tool for analysis. the main parameters, the delay and the ralative attenuation, are not included in the model explicitely, this makes impossible to use the polynomial model in any simulation work.

A noise margin analysis similar to that reported in [15], shows that the MMSE equalizer has an 8dB tolerance to noise higher than the ZF one.

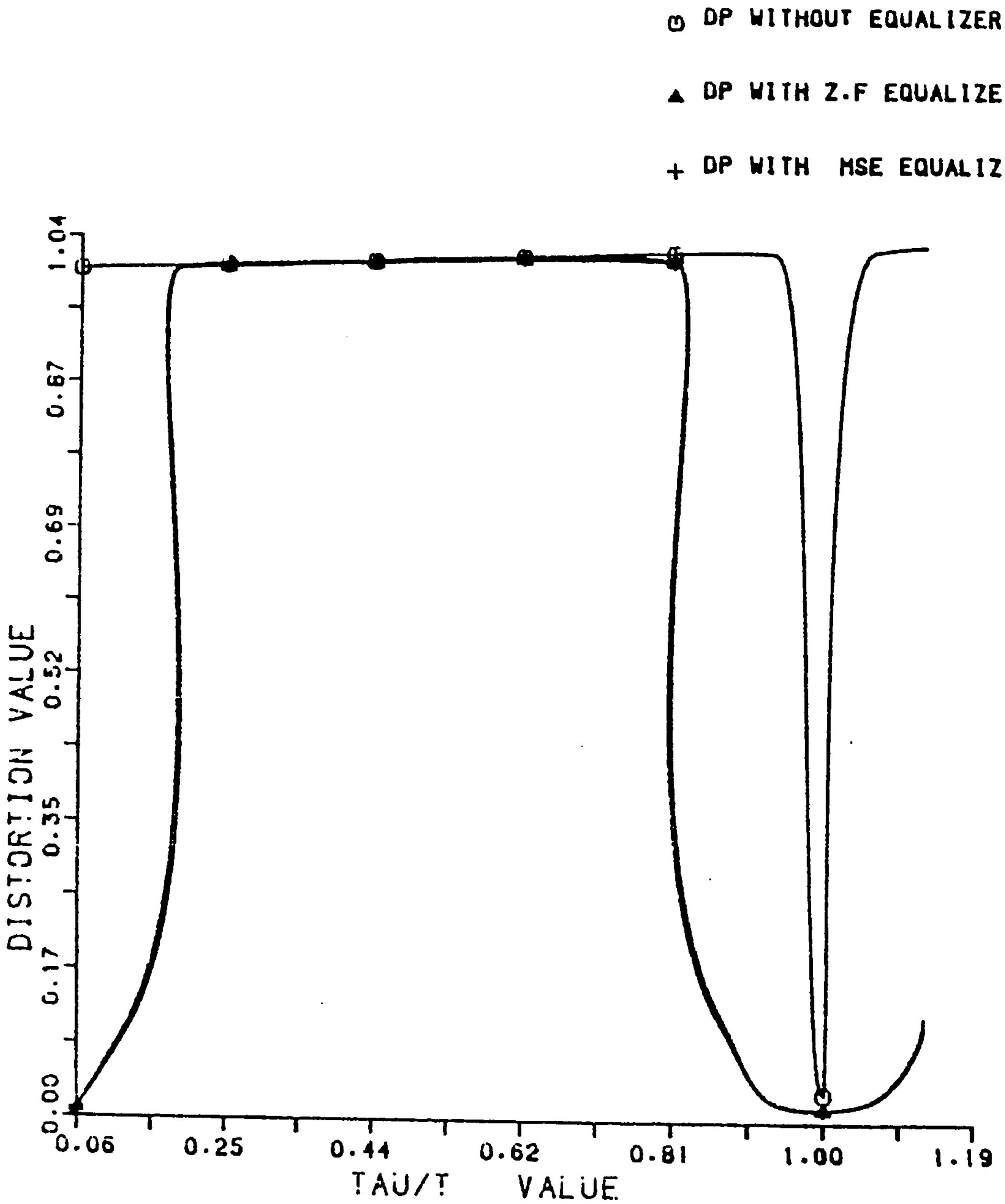


Fig 4.1 Peak Distortion variation with tau/T For the Two-Ray Model (beta= 0.1 and with a 5-tap equalizer)

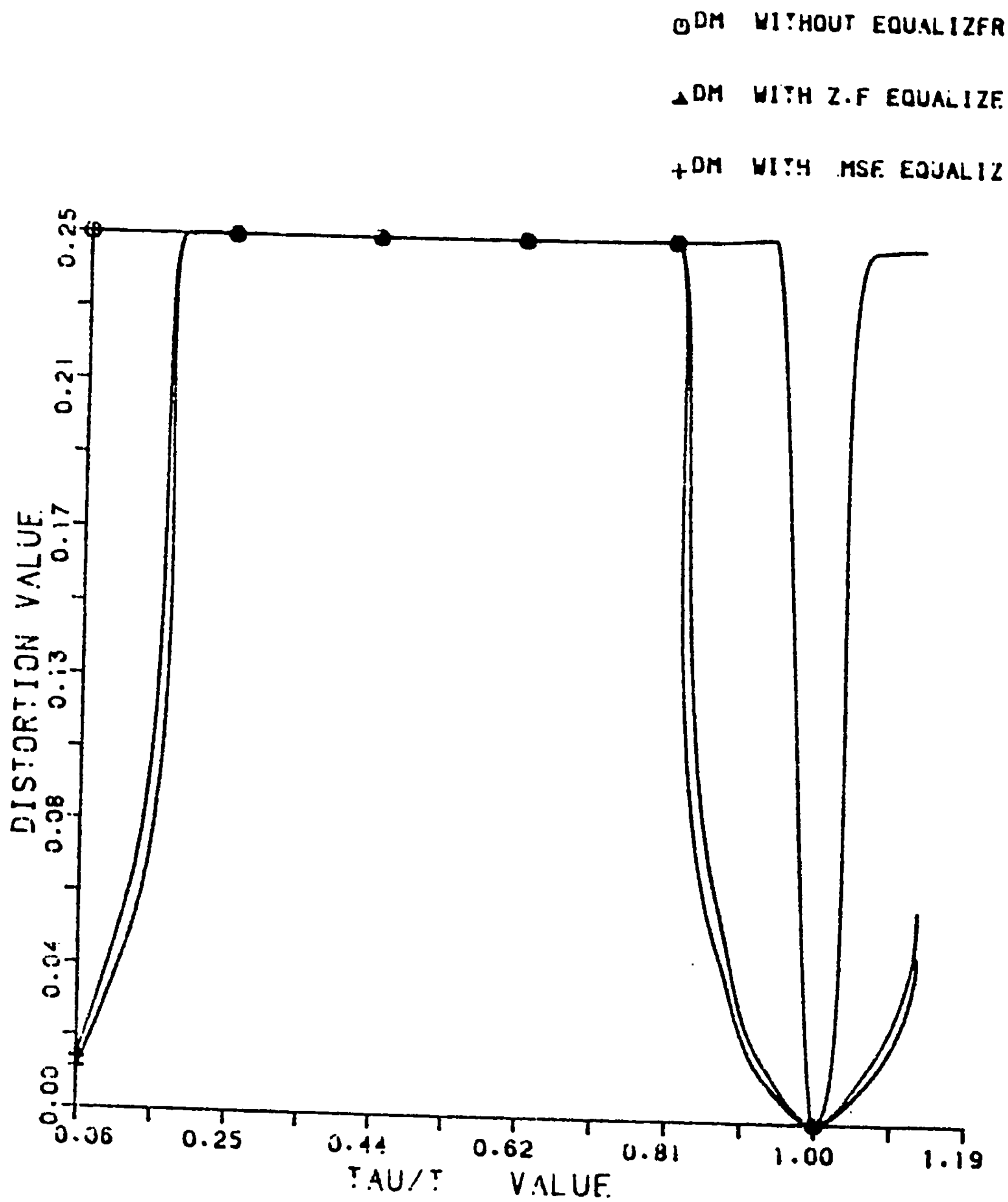


Fig 4.2 Mean Square Distortion variation with tau/T For the Two-Ray Model (beta = 0.5 and with a 5-tap equalizer)

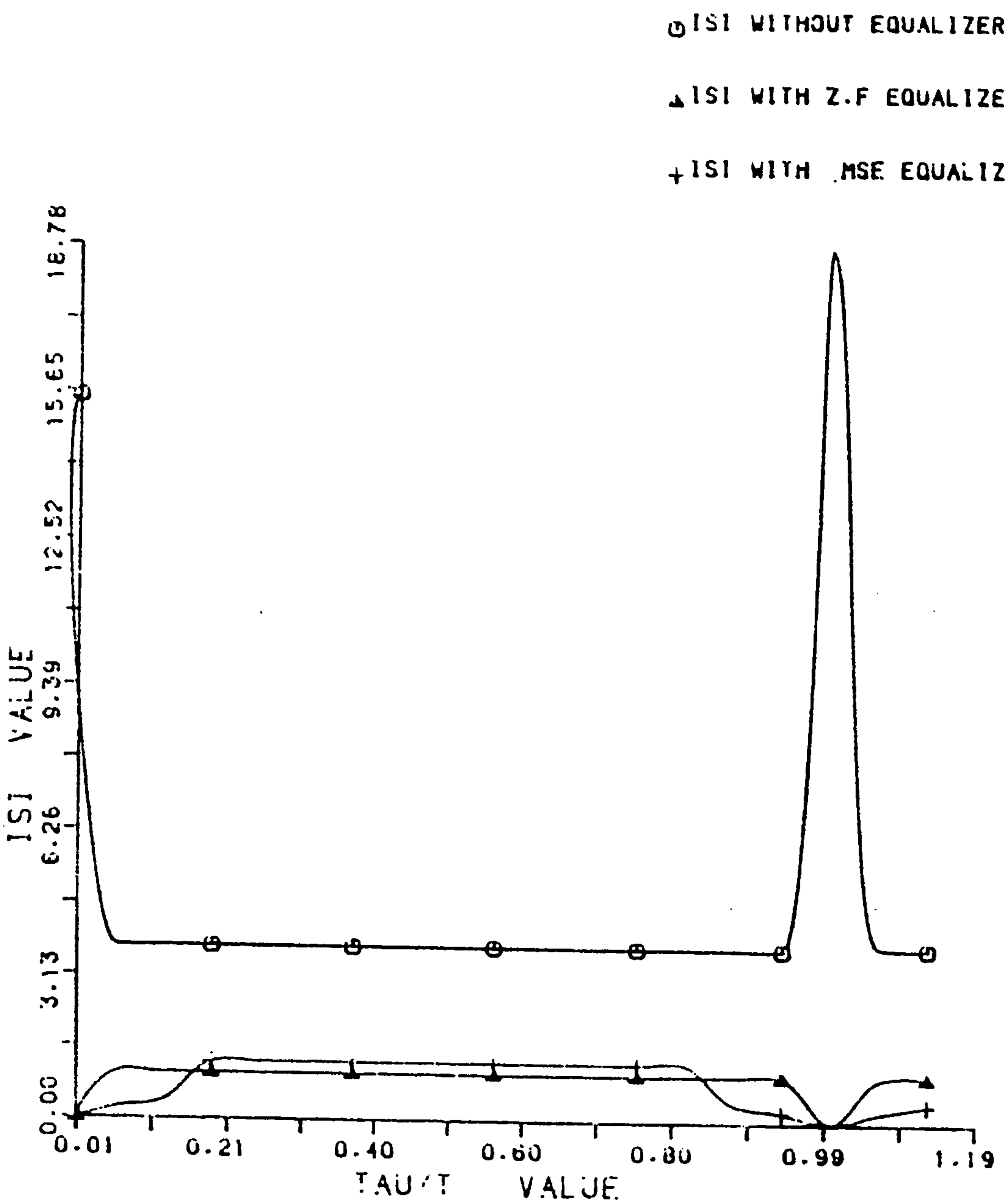


Fig 4.3 MSE or ISI variation with tau/T For the Two-Ray Model (beta = 1.0 and with a 5-tap equalizer)

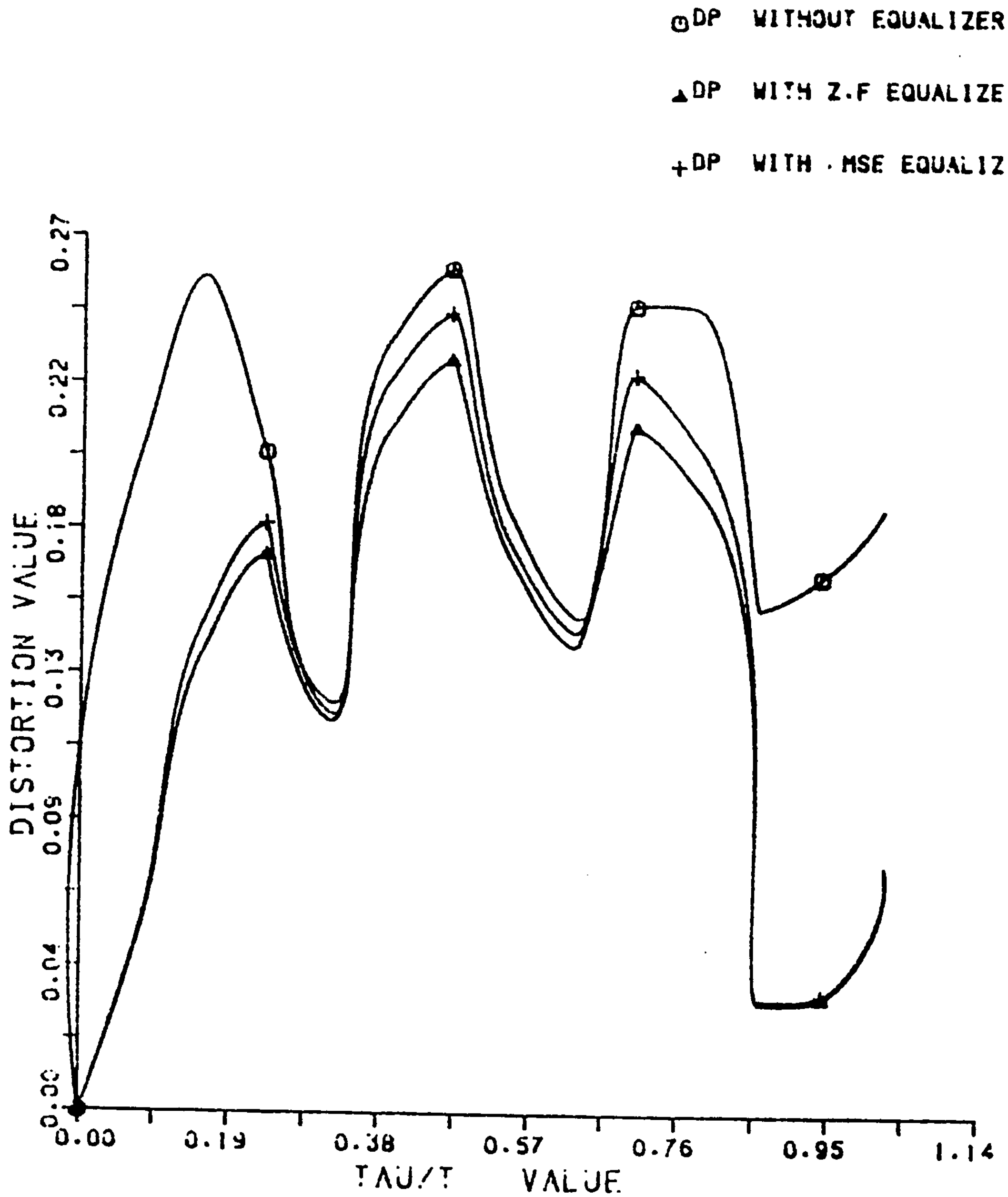


Fig 4.4 Peak Distortion variation with tau/T For the Three-Ray Model (beta = 0.1 and with a 5-tap equalizer)

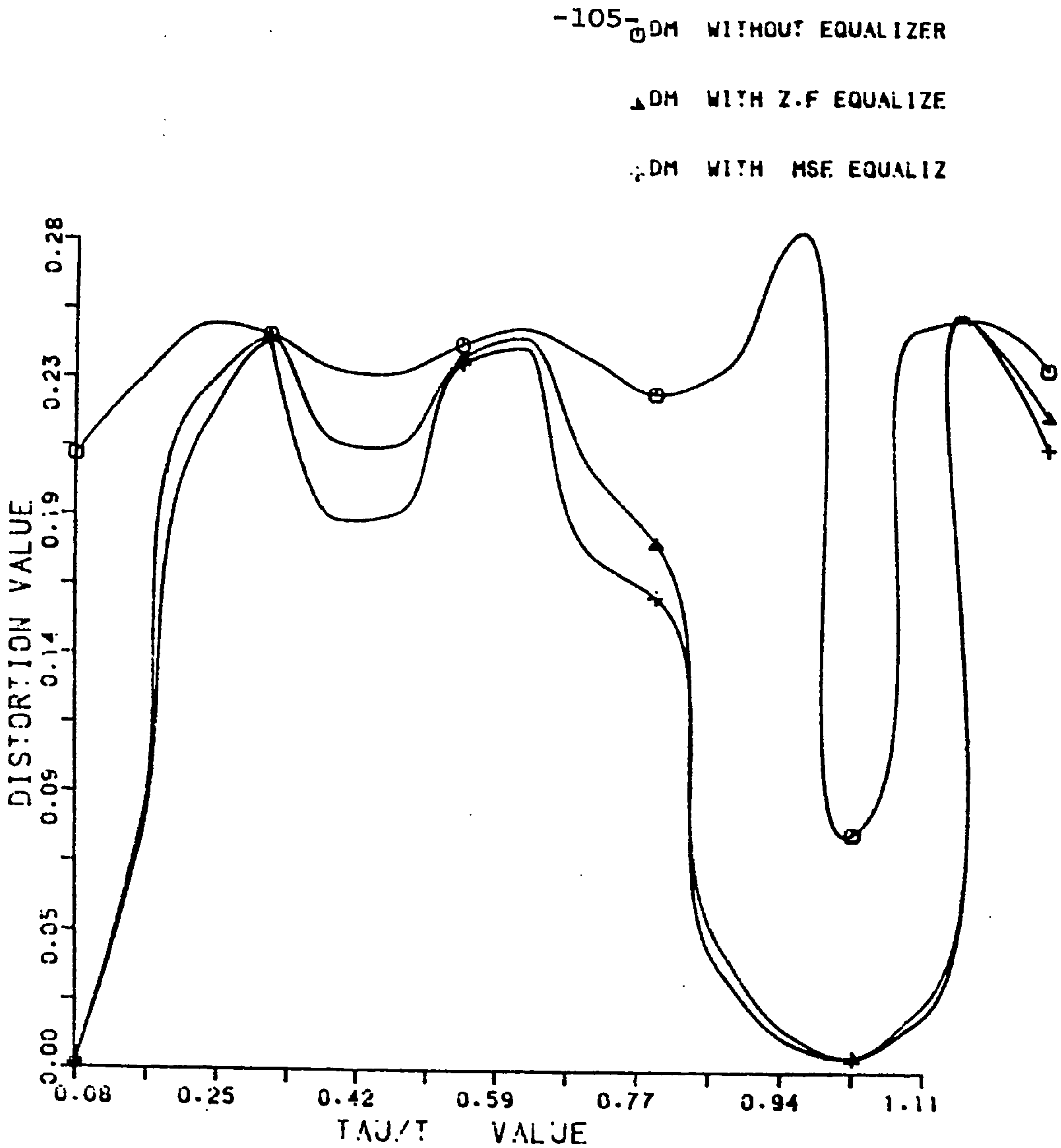


Fig 4.5 Mean Square Distortion variation with tau/T For the Three-Ray Model (beta = 0.5 and with a 5-tap equalizer)

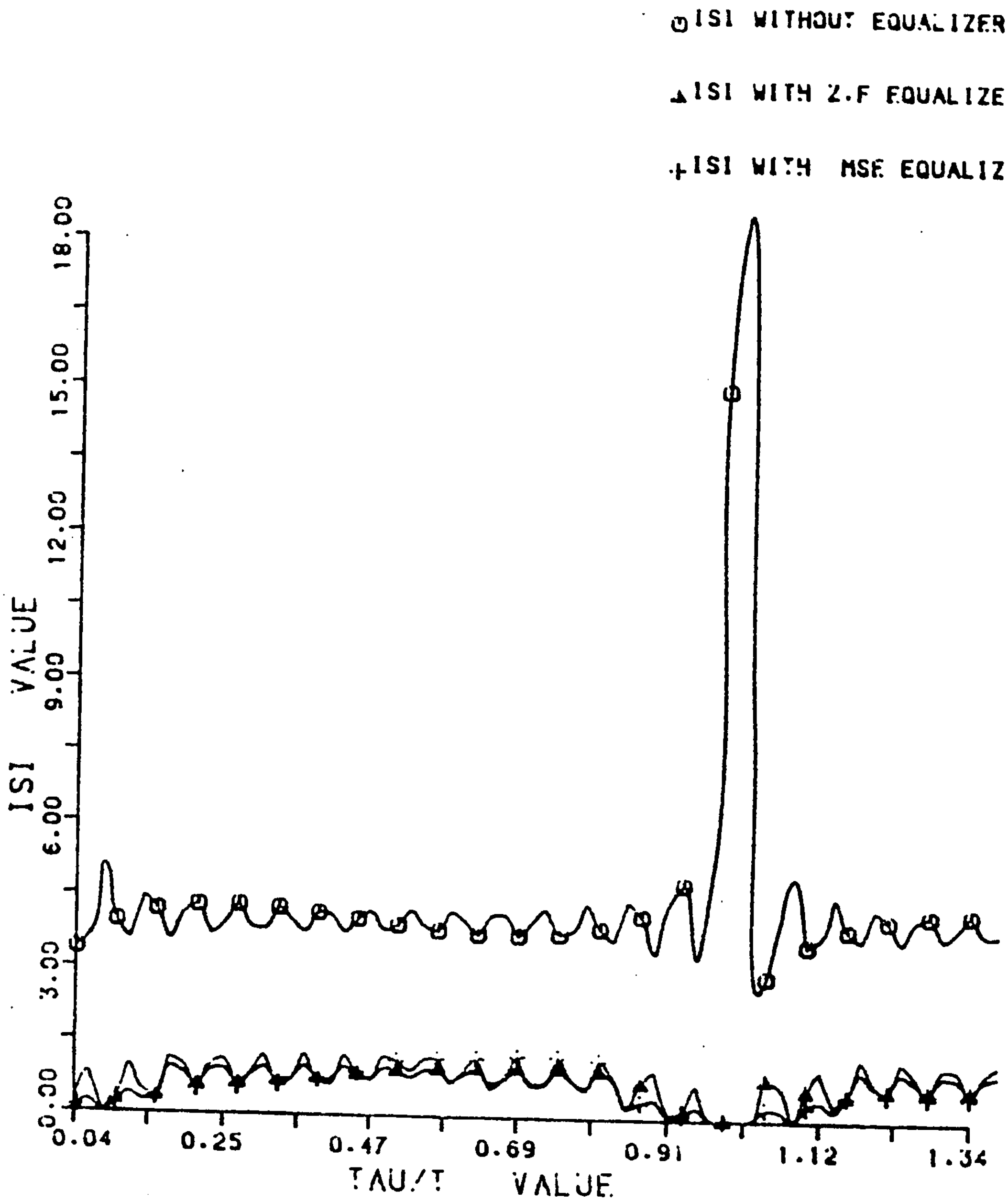


Fig 4.6 MSE or ISI variation with tau/T For the Three-Ray Model (beta = 1.0 and with a 5-tap equalizer)

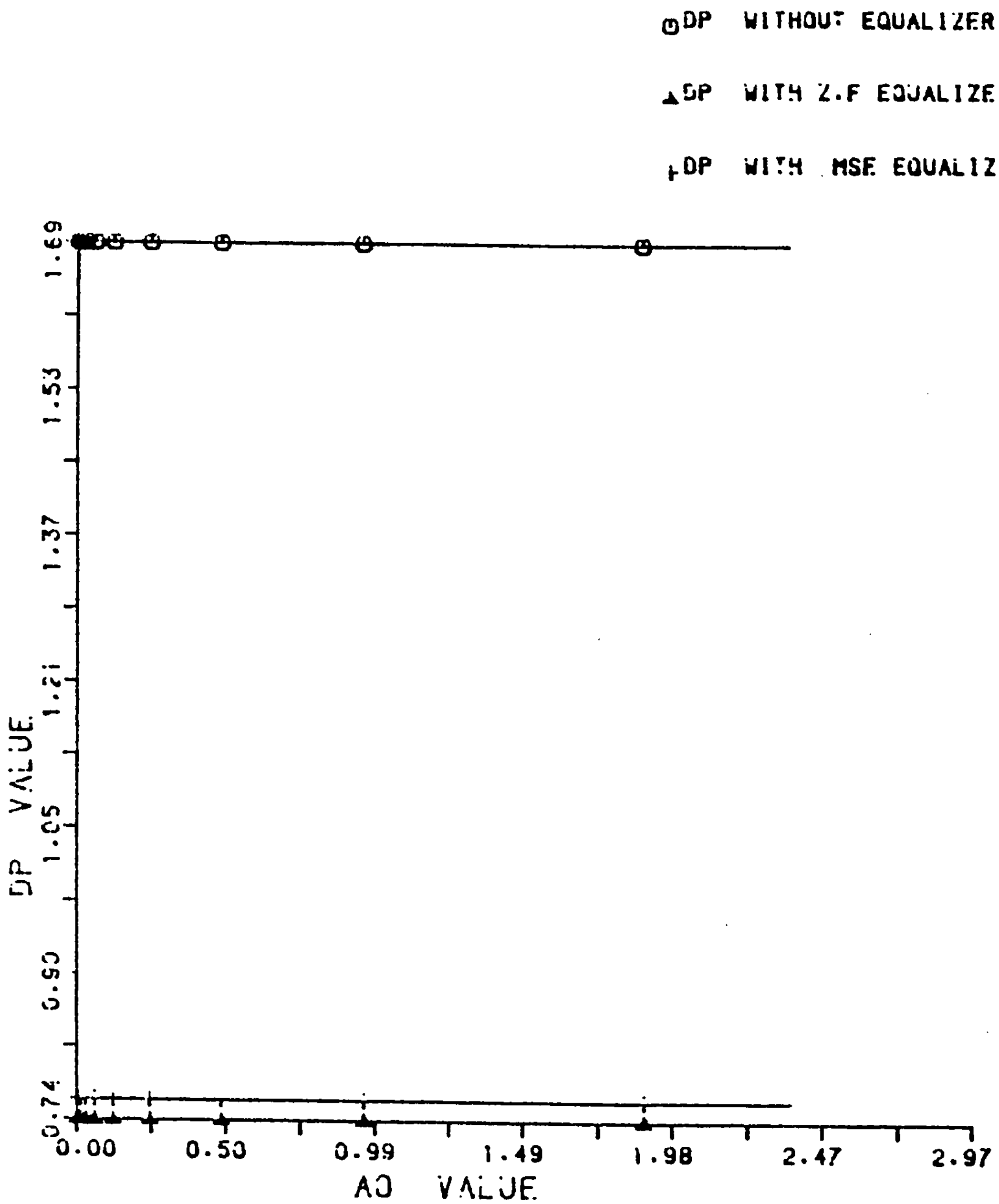


Fig 4.7 Peak Distortion variation with ao For the Polynomial Model (A1 = 0.001*ao, B1 = 0.01*ao and with a 5-tap equalizer)

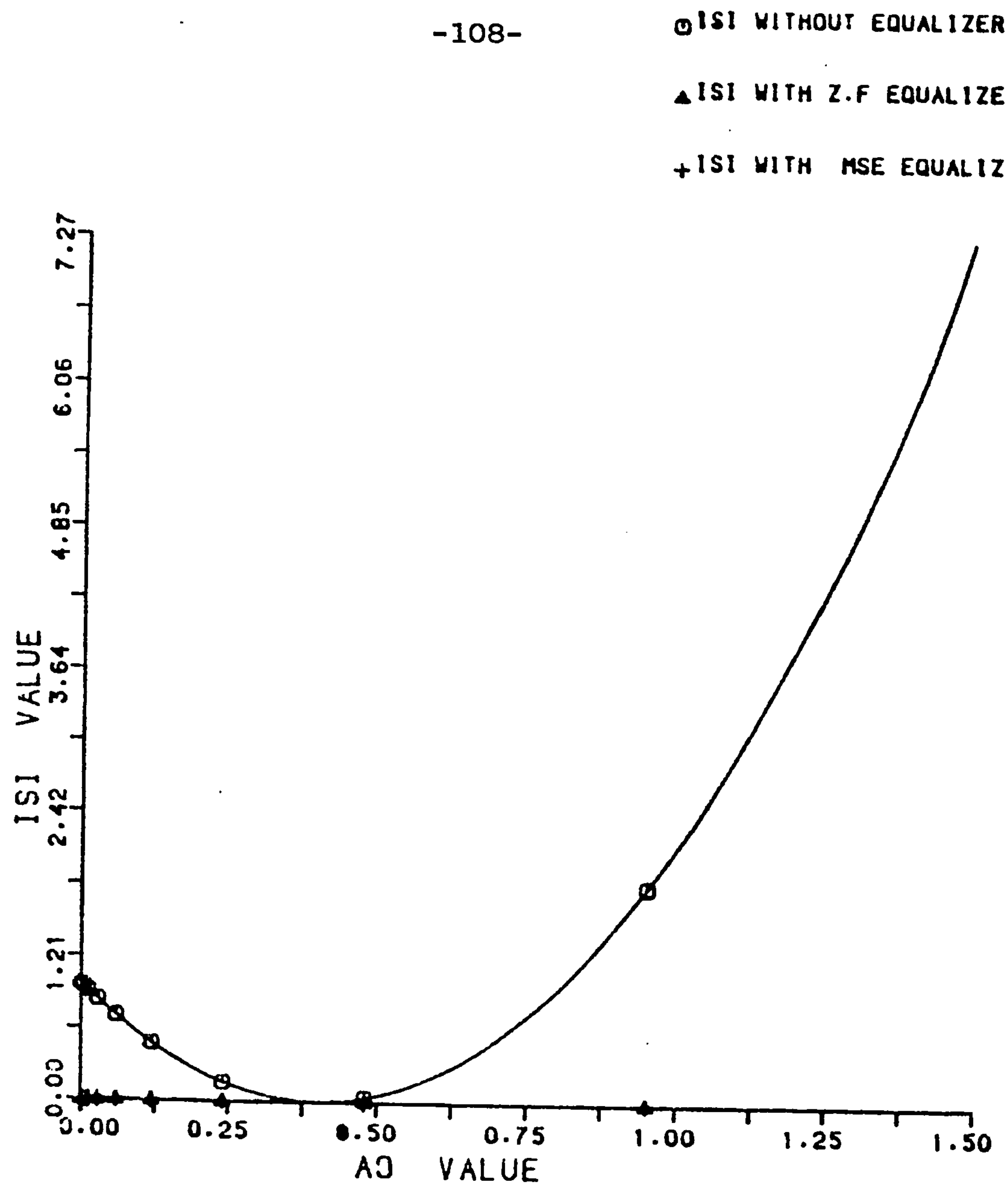


Fig 4.8 MSE or ISI variation with ao For the Polynomial Model (A1 = 0.001*ao, B1 = 0.01*ao and with a 5-tap equalizer)

CHAPTER FIVE

A HYPOTHETICAL DIGITAL MICROWAVE RADIO SYSTEM

This work on digital radio is concluded by presenting the main features and characteristics of a typical digital radio link.

The fast progress in digital technology has enabled the use of digital communication system items such as multiplexing and switching equipment. Digital radio becomes more economical for several hundred miles, however, for larger distances, analog transmission is still the economical choice [18], as shown in Fig.(5.1).

Microwave carrier frequencies encompasses the 2 up to 15 GHz range, with recommended BW transmission of 0.5% of the carrier frequency. Our next calculations will be based upon a carrier frequency of 4 GHz.

5.1: Digital radio description

Digital radio is characterized by the use of digital modulations. The approach involves modulating an intermidiate frequency (IF) carrier, typically 70 MHz, by an input data stream and then upconverting to the RF frequency range, the signal is the amplified,filtered prior to transmission. At the receiving end, the reverse process takes place. The block diagram is shown in Fig.(5.2). The specification of the different parts of the system follows in the next sections.

5.2: *Spectrum utilization efficiency*

In order to meet the different authority regulations about the spectrum utilization, high level digital modulations are employed. The modulation methods preferred now are the M-ary PSK and M-ary QAM, this is confirmed in chapter 3, however, when M-PSK is compared to M-QAM with respect to spectrum utilization efficiency, M-QAM is the more attractive as illustrated in Fig.(5.3), and table I.

Although the higher the modulation level is, the higher the spectrum utilization efficiency becomes, it is at present, difficult to use 64 or higher M-ary QAM for the following reasons:

a- The received signal spectrum suffers from high inband dispersion, which increases the sensitivity to MPF, even if space diversity and adaptive equalizer protection systems are used, as shown in Fig.(5.4).

b- The high M-ary system becomes more sensitive to the different types of interferences.

c- Equipment complexity increases and signal detection at the receivi- ng end becomes more difficult.

As a consequence, the most widely used M-ary Modulation is the 16-QAM system.

5.3: *Basis of the 16-QAM radio system*

the 16-QAM modulation conFiguration and system are shown in Fig.(5.5). The main subjects in the 16-QAM microwave radio development are:

1- The design of a high performance 16-QAM system and its equipment.

2- The correction techniques for waveform distortion due to fading.

3- The solution of various interference problems.

5.3.1: *Equipment Design Considerations*

The main items to be taken into consideration in equipment design are:

a- Filter design

The overall system filtering should be optimized to minimize the effects of intersymbol, interchannel and intersystem interferences. The filters are used in BB branch to shape the transmitted data stream spectrum, and also at the IF and RF branches.

The best results in combatting interferences is provided by the nyquist filters which, ideally, cancel ISI, but due to their difficult design sub-optimum filters, such as Butterworth and chebyshev fillters, are designed and employed. The filter roll-off factor plays an important role in determining the required BW, the lower the roll-off factor, the lesser the needed BW becomes and the higher the spectrum efficiency becomes, however, the extremely more difficult to design the filter is. A trade-off can be achieved by a roll-off factor of about 0.5, which requires a 50% excess BW to the Nyquist one.

b-Repeater Consideration

The repeater is essential in the Microwave link to regenerate the transmitted data stream, it is installed nearly every 50 Km. A block diagram for a 16-QAM repeater is depicted in Fig(5.6). Most repeaters are equipped with space diversity and adaptive equilizer arrangment to correct waveform distortion.

5.3.2: Multipath Countermeasures

a-Character of microwave routes

Fading occurrence probability depends not only on the hop length and frequency, but also on the terrain topography, this is revealed explicitely on the form of the factor T M . The terrain can be water, mountains or plains, the probability of fading becomes particularly large over water areas. However, the reflected waves from water surfaces has larger delay than those reflected by the inversion layers, which causes larger system outage.

b- Space diversity

Space diversity is one of the most effective methods for combatting Frequency selective fading, it can minimize inband amplitude dispersion. Conventionally, space diversity arrangment uses maximum amplitude combiner " in-phase combiner", which cannot remove completely

the inband dispersion, however, the new minimum dispersion combiner minimizes sufficiently inband dispersion leading to waveform distortion raduction. The two combiners and their performances are illustrated in Fig.(5.7).

c- Adaptive Equilizer

As mentionned in the earlier chapters, adaptive equilization technique is adopted to correct the amplitude and delay distortions. The combined improvement factor of space diversity and an adaptive equalizer is more than 100.

5.3.3: *System performance with MPF*

Digital radio designer has adopted M-QAM mostly in their systems. When compared to other modulation schemes, it shows superiority in system availability and spectrum efficiency. As an example, the bit rate required for a 4 KHz voice signal is 64Kb/s. At 4GHz, the rule of 0.05% Bw gives 20 MHz of BW, theoritically, in this BW, 16-QAM which has a spectral efficiency of 4b/s/Hz or a symbol/s/Hz, enables to transmit 20MHz * 4b/s/Hz = 80 Mb/s. if we consider a BB roll-off factor of 0.5, that is an excess BW of 50%, the practical transmission rate is 80Mb/s/1.5 = 53 Mb/s or nearly 833 voice signal of capacity.

a- MPF fading effects

The effects of MSF is drastic since Pe falls drastically to low values, and oscillates during FSF depicting destructive and constructive behaviour. This is illustrated in tables(II), App V. and shown in Figs(5.8-9)

b-Equalizer implementation **:**

The performance of ZF or MMSE equalizer when used at the receiving end has the forllwing observations:

1- Complete reduction of ISI when the delay is of one bit duration either by ZF or MMSE equalizer.

2- The ZF is effective in removing Dp more than the MMSE, as the former is based upon minimizing Dp.

3- The oscillations picture of ISI or Dp terms is also present, that is their values are varying between two extremes. Fig(4.6).

4- Comparing between tables(5.3) and (5.6), the three-ray model offer better results in which the ocsillations of ISI with the fraction delay are illustarated.

5- From table(5.6), we can see that MMSE is more effective in equalizing the ISI peaks than the ZF one which equalizes more the minima, however, the differences are very small, but both reduce ISI by nearly 75%.

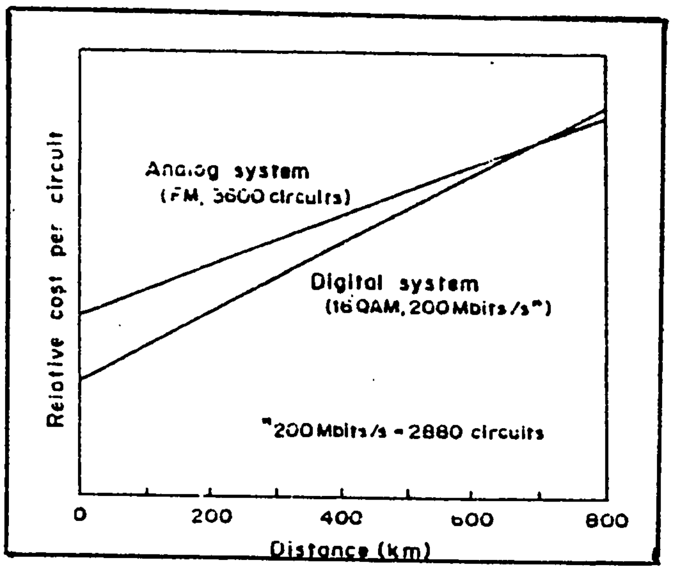


Fig 5.1 Analog vs Digital system ralative cost

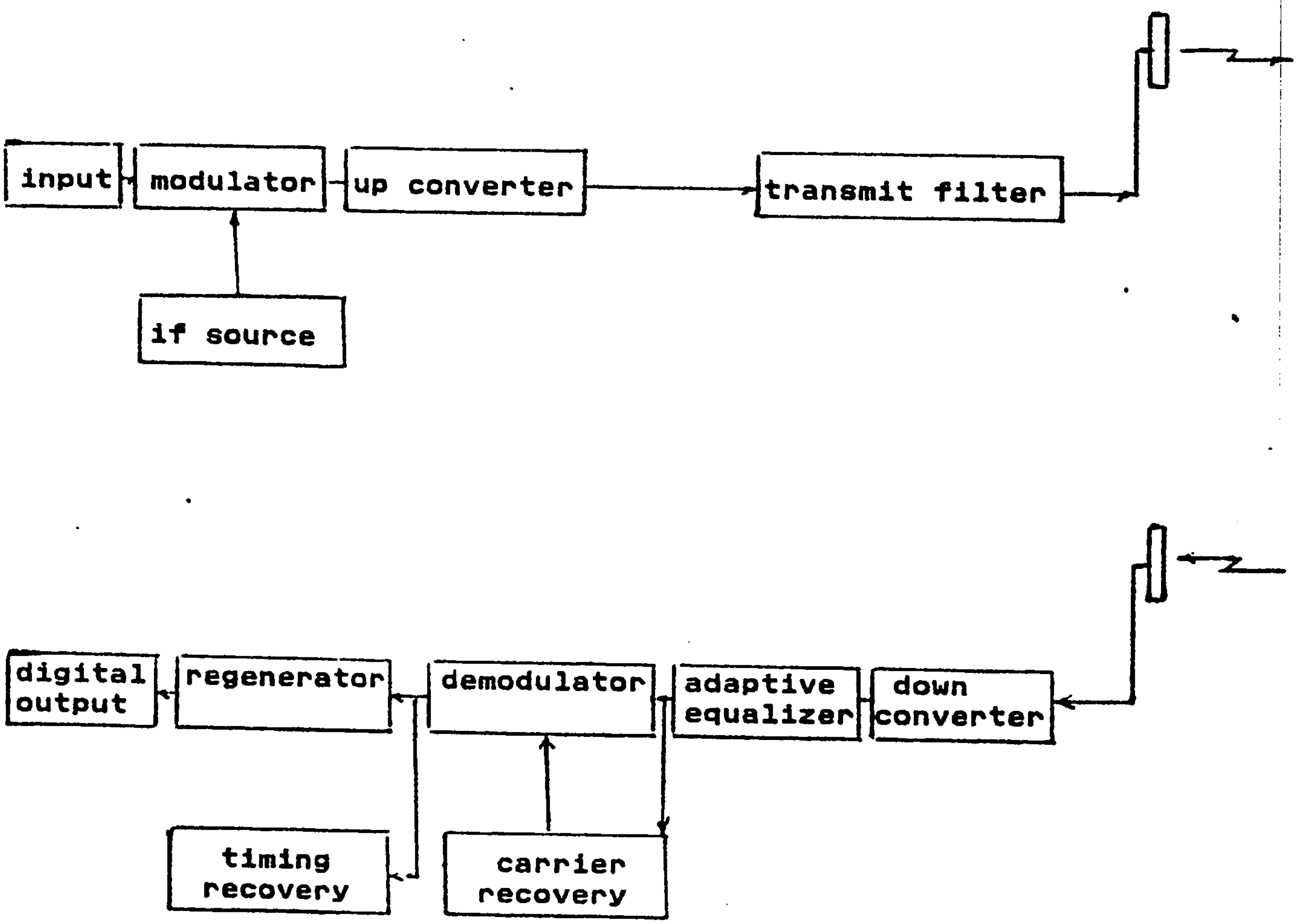


Fig 5.2 Digital radio block diagram

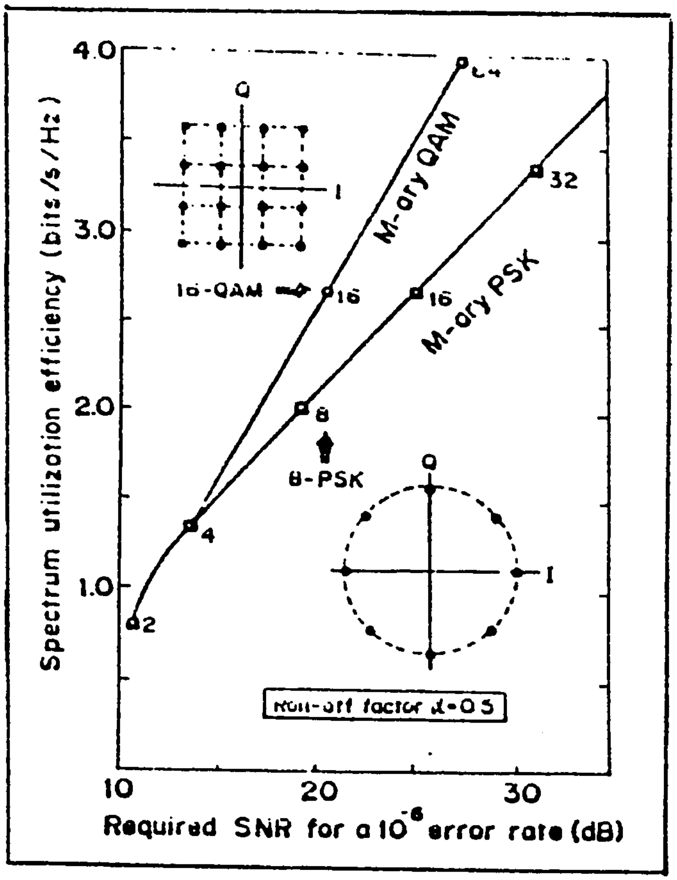


Fig 5.3 M-QAM and M-PSK spectrum utilization efficiency Comparison

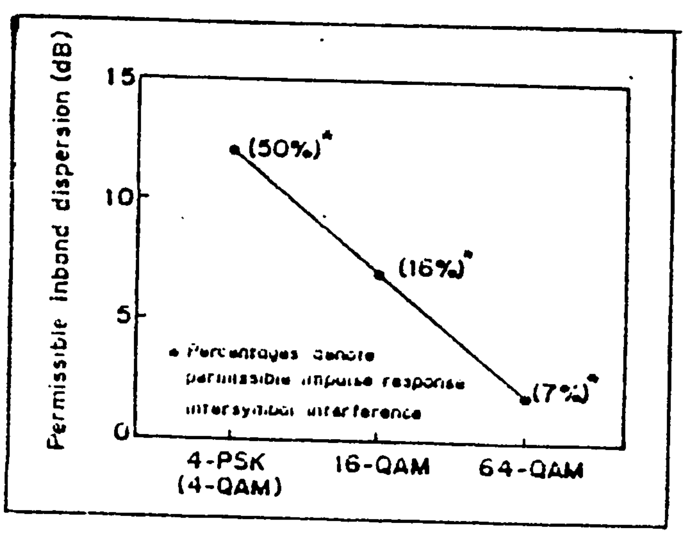


Fig 5.4 Permissible inband-dispersion For M-QAM

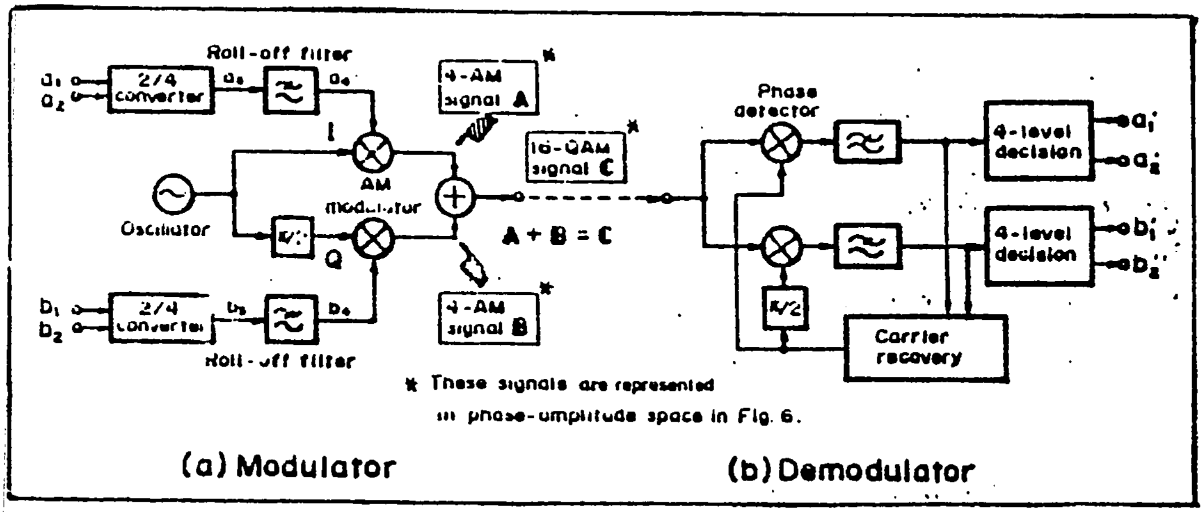


Fig 5.5 16-QAM system configuration

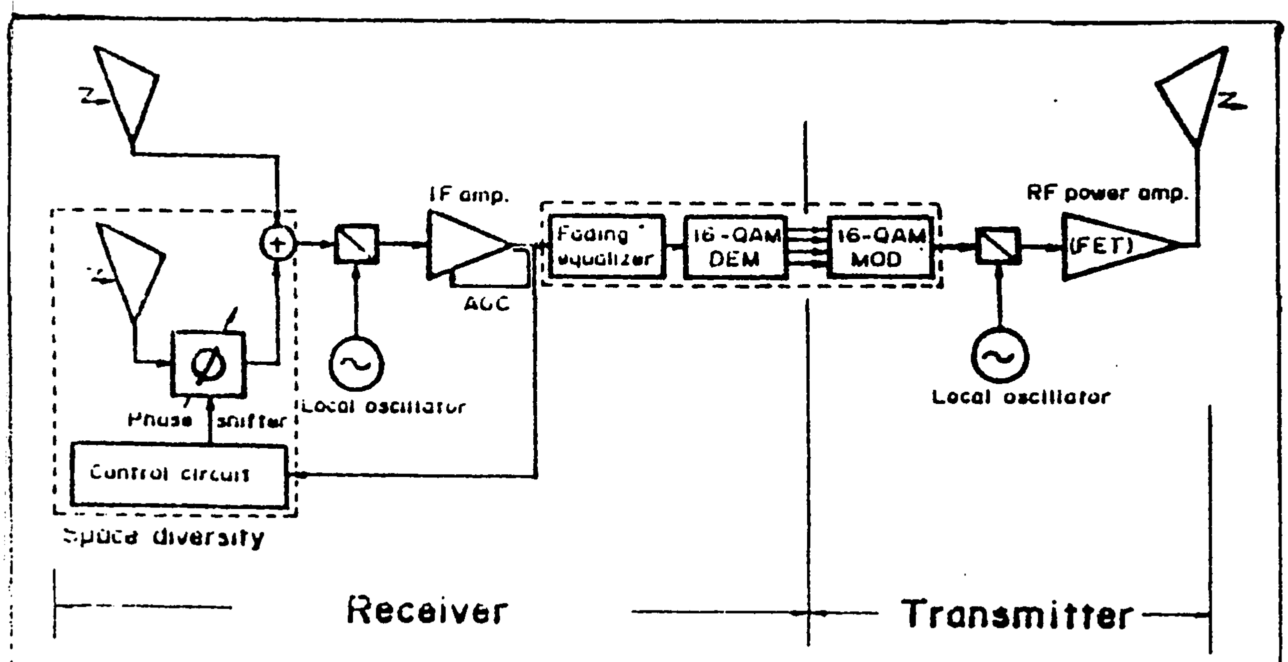


Fig 5.6 16-QAM repeater block diagram

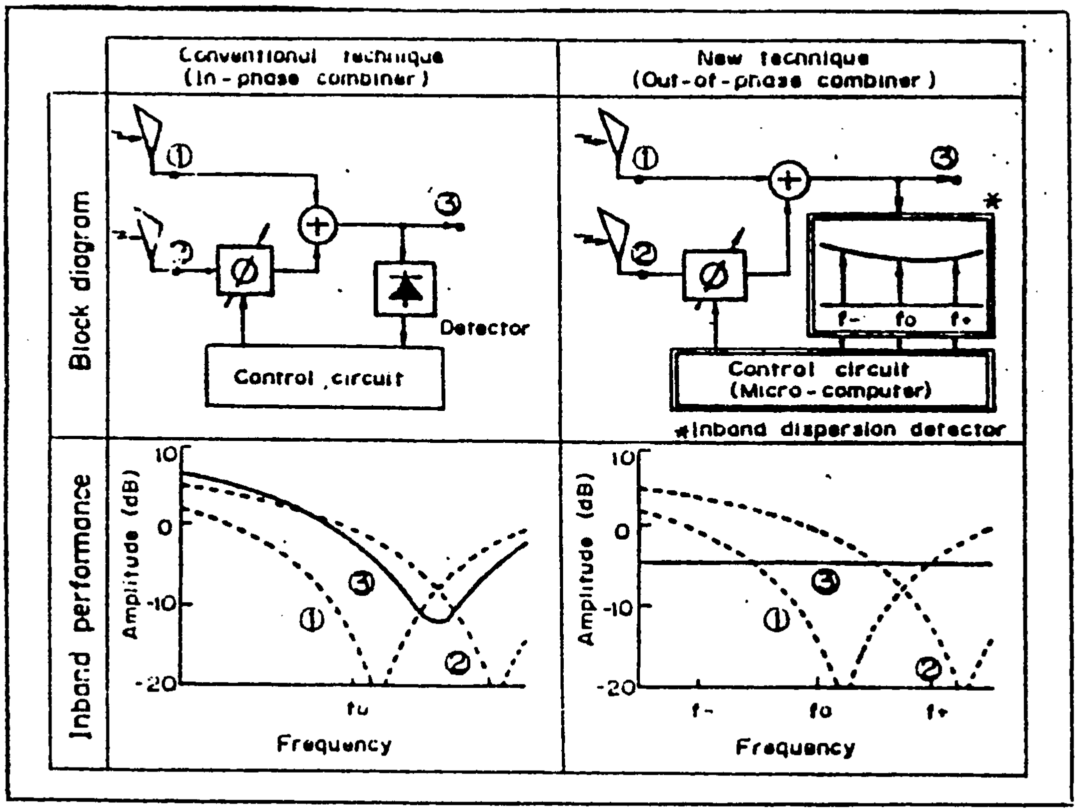
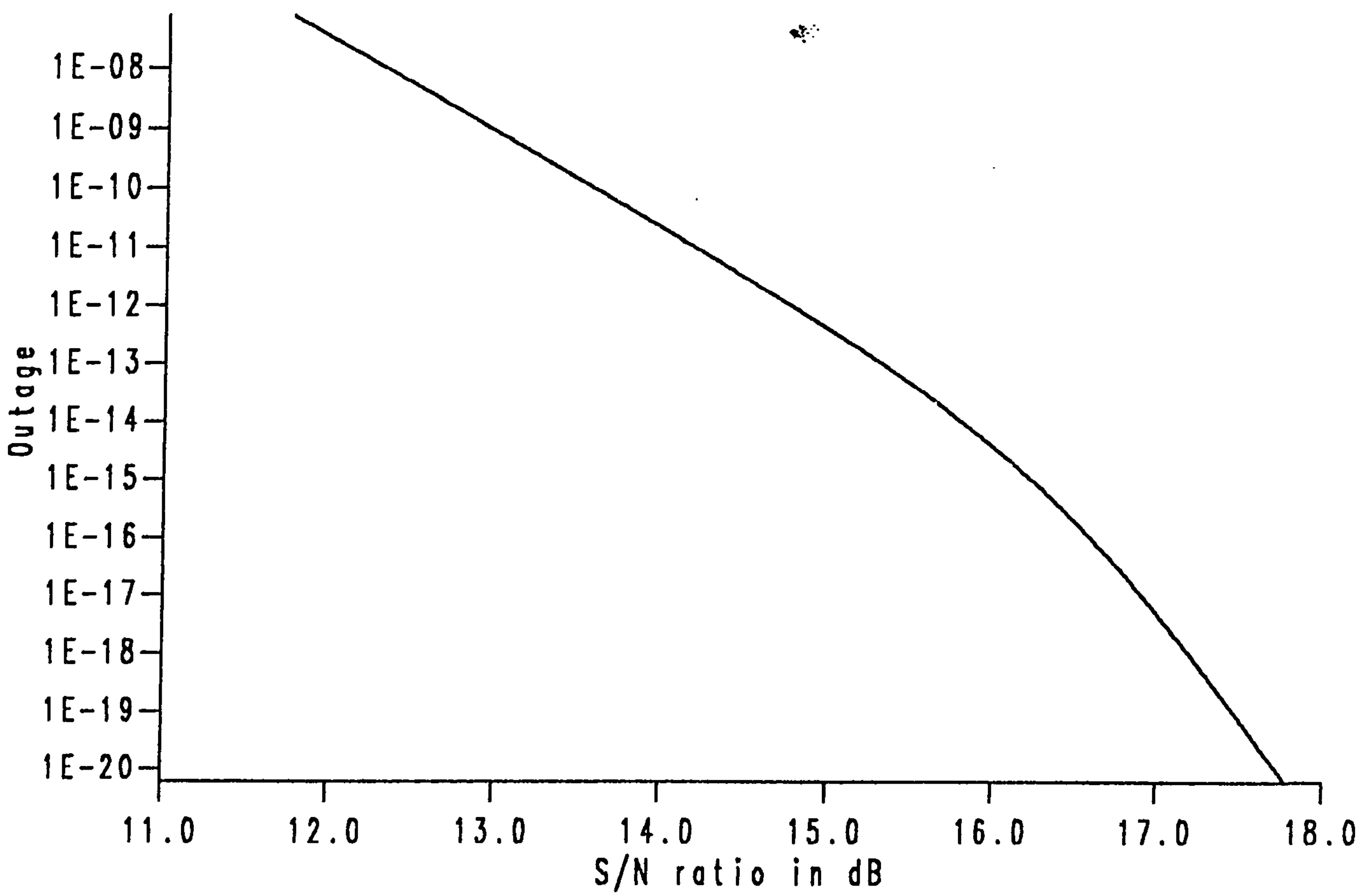
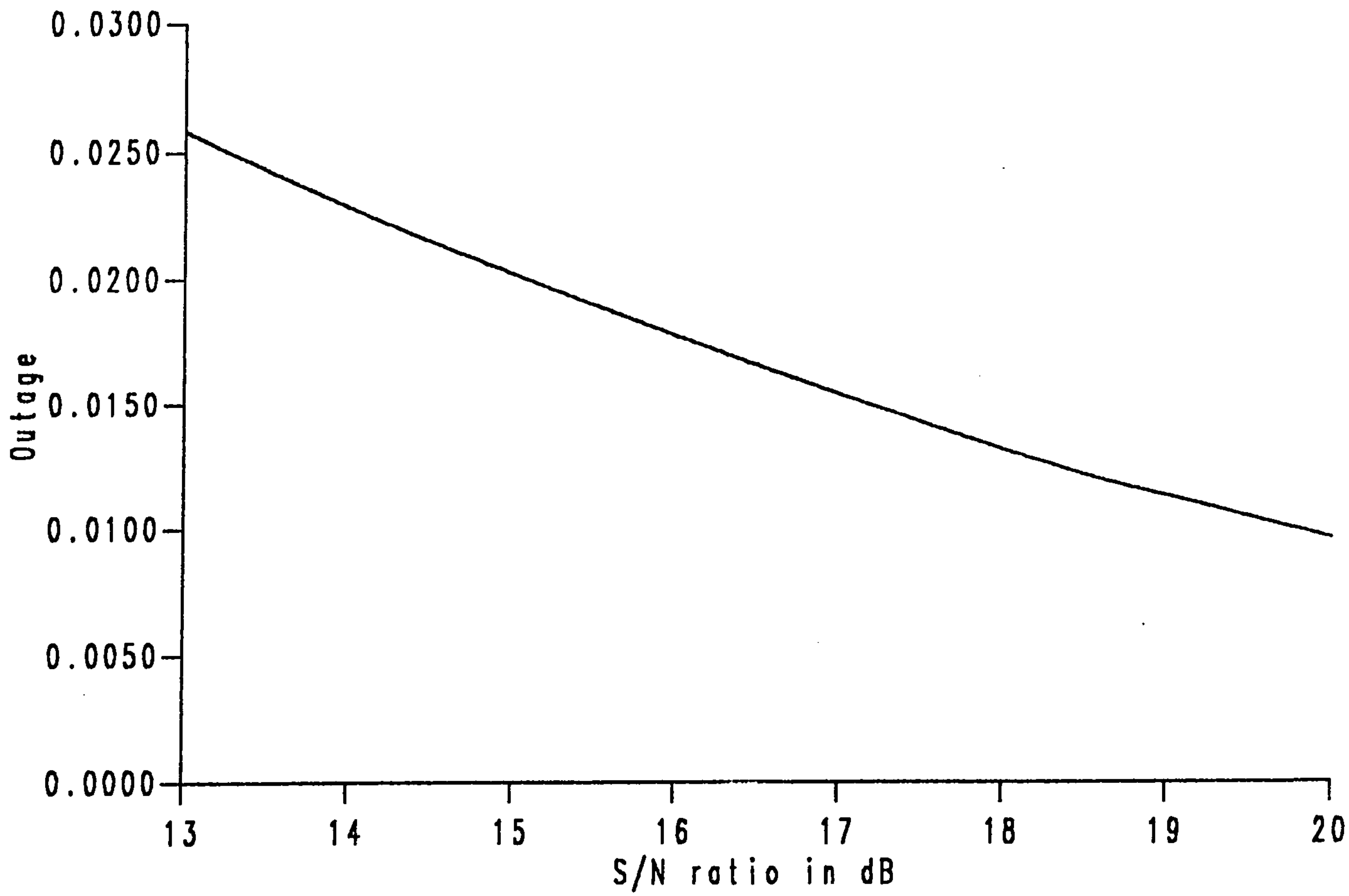


Fig 5.7 In-phase and minimum dispersion combiners performances

Fig(5.8): Outage versus S/N for 4-QAM Scheme



Fig(5.9): Outage versus S/N for 16-QAM Scheme



CONCLUSION

1- The Flat Fading affect the communication system beyond the Gaussian noise impact, and a small relative amplitude value can put a burden up to 2 dB.

2- Frequency Selective Fading contributes with a constructive or a destructive role to the system, the performance can be enhanced or deteriorated dramatically.

3. the delay increment palys a major role in FSF , however, the resulting BER fucluates aroud a mean value, but increasing the carrier frequency enhance the system availability.

4- The Zero Forcing and Minimum Mean Square Error Equalizers succed in removing ISI more than peak and mean square distortions.

5- The Two-Ray and Three-Ray models can adequately used to model MPF to evaluate system performance, but the polynomial model has a major weakness in the fact that the delay and relative amplitude of the interfering rays are not included explicitely.

Suggestions for further Research

- The Equalizer set is applied at the BB section where the stream is binary, it can be applied at the quadrature or in-phase channels, where M&half. levels are to be processed.

- The MPF is analysed through the two-ray model. The same work can be done with the three-ray model.

- The equalizer ste can be composed of a MMSE equalizer and a processor to adapt the tap coefficients to handle the continuous variation of the parameters.

- The transmitting and receiving filters are considered ideal, the work can be done with practical ones.

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APPENDIX I

Error Probability Expressions for Coherent Modulations

* *Amplitude- Shift Keying(ASK)*

The signal states are given by:

$$s(t) = \begin{cases} 0 & 0 < t < T \\ A\cos(w_o t) & 0 < t < T \end{cases}$$

Where w_o : the carrier frequency

T : the bit duration

The corresponding probability of error

$$P_e = \frac{1}{2}\, erfc \sqrt{\frac{z}{2}} \qquad \text{(I-1)}$$

where, $z = \frac{E}{\eta}$: the average received S/N ratio

E : the average signal energy

$\frac{\eta}{2}$: the noise power spectral density

* *Phase-Shift keying (PSK)*

$$s(t) = \left[\begin{array}{ll} A\cos(w_o t) & 0 < t < T \\ -A\cos(w_o t) & 0 < t < T \end{array} \right.$$

and

$$P_e = \frac{1}{2}\, erfc\, \sqrt{z} \qquad \text{(I-2)}$$

** Frequency-Shift Keying (FSK)*

$$s(t) = \left[\begin{array}{ll} A\cos(w_o t) & 0 < t < T \\ A\cos(w_o + \Delta w)t & 0 < t < T \end{array} \right.$$

Where $w_o = 2\pi N/T$, and $\Delta w = 2\pi M/T$

M , N are integers, hence

$$P_e = \frac{1}{2}\, erfc\, \sqrt{\left(\frac{z}{2}\right)} \qquad \text{(I-3)}$$

Probability Expressions For Non-Coherent Modulation Schemes

** Differential PSk (DPSK)*

$$P_e = \frac{1}{2}\, e^{-z} \qquad \text{(I-4)}$$

** N-ASK*

$$P_e = \frac{1}{2} e^{-\frac{z}{2}} \left[\frac{1}{2} + \frac{1}{\sqrt{2\pi z}}\right] \qquad (I-5)$$

Where, $z = \dfrac{A^2}{4\eta B_T}$

B_T = the bandpass filter at the demodulator

* *N-F S K*

$$P_e = \frac{1}{2} e^{-\frac{z}{2}} \qquad (I-6)$$

with, $z = \dfrac{A^2}{2\eta B_T}$

APPENDIX II

Flat-Fading Effect On PSK Modulation

In case of PSK, we have at the receiving end

The received signal

$$y(t) = s(t) + \beta s(t - \tau_m) + n(t) \qquad \text{(II-1)}$$

Where s(t) is the received direct-path component given by

$$s(t) = A\, d(t) \cos w_o t$$

and, $\quad d(t)$ = the data stream

β, τ_m are the relative attenuation and delay of the interfering ray.

Flat-fading occurs when

$$\frac{\tau_m}{T} \simeq 0, \text{ so } d(t-\tau_m) \simeq d(t)$$

In this case, $w_o \tau_m$ is uniformly distributed in $(-\pi, \pi)$.

This is true if $w_o \tau_m$ fluctuates much faster than the tracking loop time constant. Otherwise, the term has the same effect as the AWGN.

Once again, the optimum filter is considered at the receiving end, and the matched filter is consisting of a coherent phase detector.

$$y(t) = A\ d(t)\ cos(w_o t) + \beta(t - \tau_m)cos(w_o(t - \tau_m)) + n(t)$$

d(t) is a data stream of ±1

Writing n(t) in terms of its quadruture components, $n_c(t)$ and $n_s(t)$

$$y(t) = [A\ d(t)+n_c(t)]cosw_o t+\beta d(t-\tau_m)cosw_o(t-\tau_m)-n_s(t)sinw_o t$$

Thus

$$x(t) = 2\ y(t)\ cosw_o t$$

Ignoring the double frequency terms ,as they will be removed at the output of integrator.

$$x(t) = A\ d(t) + n_c(t) + \beta A\ d(t - \tau_m)cosw_o\tau_m$$

and Keeping in mind that d(t) $\simeq d(t - \tau_m)$

then $$x(t) = Ad(t)[1 + \beta cos(\phi)] \qquad (II-2)$$

where $\phi=w_o\tau_m$ is uniformly distributed given by

$$p(\phi) = [\ \frac{1}{2}\pi \qquad \phi\varepsilon(-\pi,\pi)$$

$$0 \qquad elsewhere$$

Now, keeping ϕ constant

$$E(x(t)^2) = A^2(1+\beta\cos\phi)^2 + E(n_c(t)^2)$$

The S/N ratio is given by

$$z = \frac{A^2[1 + \beta cos(\phi)]^2}{E(n_c(t)^2)}$$

where $\qquad E(n_c(t)^2) = \eta B_T$

For a PSK system $B_T \simeq 2r_b$, Thus

$$z' = \frac{A^2[1 + \beta cos(\phi)]^2}{2\ N_o}$$

$$= z\ (1 + \beta\cos\phi)^2 \qquad \text{(II-3)}$$

From (I-3)

$$P_e = \frac{1}{2}\ erfc\sqrt{z}$$

then with the flat fading term

$$P_e/_\phi = \frac{1}{2}\ erfc\ [\sqrt{z}\ (1+\beta\cos\phi)] \qquad \text{(II-4)}$$

For $z >> 1$, and using the following approximation

$$erfc(u) = \frac{\exp^{-u^2}}{\sqrt{u\pi}}, \qquad u >> 1$$

$$P_{e/\phi} = \frac{e^{-z} \ (1 + \beta\cos\phi)^2}{2(\pi z)^2(1 + \beta\cos\phi)}$$

if $|\beta| << 1$, and $(1 + \beta\cos\phi)^m \simeq 1+m\beta\cos\phi$

then

$$P_{e/\phi} = e^{-z} \ (1 + 2\beta\cos\phi) \ \frac{(1 - \beta\cos\phi)}{2\sqrt{\pi z}}$$

The average probability is finally

$$P_e = [I_o(2z\beta) + \beta I_1(2z\beta] \ \frac{e^{-z}}{2\sqrt{\pi z}} \qquad \text{(II-5)}$$

$$z >> 1 \ , \ |\beta| << 1$$

where $I_o(x)$ and $I_1(x)$ are modified bessel fuctions of the first kind, given by

$$I_o(x) = \frac{1}{2}\pi \int_{-\pi}^{\pi} e^{-2z\beta \ cos(\phi)} \ d\phi$$

$$I_o(x) = \frac{1}{2}\pi \int_{-\pi}^{\pi} -\cos\phi \ e^{-2z\beta \ cos(\phi)} \ d\phi$$

APPENDIX III

The decoding strategy for 16-QAM under FSF is as following:

let Y(Ts) be the output of the detector per in one channel, and D(Ts) the correct decision in terms of bits.

$$Y(Ts) = V(Ts) + N$$

where $$N = \int_{0}^{Ts} n(t) \cos(w_o t)\, dt$$

so if

$$Y(Ts) > 2ATs \longrightarrow D(Ts) = 11$$
$$0 < Y(Ts) < 2ATs \longrightarrow D(Ts) = 01$$
$$-2ATs < Y(Ts) < 0 \longrightarrow D(Ts) = 10$$
$$Y(Ts) < -2ATs \longrightarrow D(Ts) = 00$$

hence, if the string 11 was transmitted

$$P(E/1111) = P[\ Y < 2ATs\ ]$$
$$= P[\ V_{++++} + N < 2ATs\ ]$$

but $$V_{++++} = 3ATs\,(1 + \delta)$$

$$P(E/1111) = P[\ ATs(1 + 3\delta) + N < 0\ ]$$

$$= 1/2 \quad \text{erfc} \quad [\quad ATs(1 + 3 \delta) \frac{1}{\sqrt{2}\sigma_N}]$$

where

$$\sigma^2_N = \int_0^{Ts} n(\tau)\, n(t)\, \cos w_o t\, \cos w_o \tau \, dt\, d\tau$$

$$= \eta \frac{Ts}{2}$$

so $\quad P(E/1111) = \frac{1}{2}\, erfc[\sqrt{z}(1 + 3\delta)]$

where

$$z = A^2 \frac{Ts}{2\eta}$$

$$P(E/0111) = P \,[\quad Y < 2ATs \quad]$$

$$= P\, [\, V_{-+++} + N < 2ATs \,]$$

but $\quad V_{-+++} = 3ATs(1+\delta) - [3A(1+\delta) - A(3+\delta)]\, \tau_m$

$$= ATs \,[\, 3 \,(\, 1 \, + \delta) - \delta(\frac{\tau_m}{Ts})]$$

$$P(E/0111) = P\, [\, ATs\, [\, 1 + 3\delta - \delta(\frac{\tau_m}{Ts})] + N < 0 \,]$$

$$= \frac{1}{2}\, erfc[\sqrt{z}\, (1 + 3\delta - \delta(\frac{\tau_m}{Ts})]$$

$$P(E/1011) = P \,[\quad Y < 2ATs \quad]$$

$$= P\ [\ V_{+-++} + N < 2ATs\ ]$$

but $$V_{+-++} = 3ATs\ (1+\delta) - [3A(1+\delta) - A(3-\delta)]\ \tau_m$$

$$= ATs\ [\ 3\ (\ 1 + \ \delta) - 2\delta(\frac{\tau_m}{Ts})]$$

$$P(E/1011) = \frac{1}{2}\ erfc[\sqrt{z}\ (1 + 3\delta - \delta(\frac{\tau_m}{Ts})]$$

Finally

$$P(E/0011) = P\ [\ \ Y < 2ATs\ \ ]$$

$$= P\ [\ V_{--++} + N < 2ATs\ ]$$

but $$V_{--++} = 3ATs\ (1+\delta) - [3A(1+\delta) - 3A(1-\delta)]\ \tau_m$$

$$= ATs\ [\ 1 + 3\ \ \delta - 3\delta(\frac{\tau_m}{Ts})]$$

$$P(E/0011) = \frac{1}{2}\ erfc[(\sqrt{z})\ (1 + 3\delta - 3\delta(\frac{\tau_m}{Ts})]$$

Similarly , if the string 01 was transmitted

$$P(E/0101) = P\ [V_{-+-+} + N < 0 \ \text{ or } V_{-+-+} + N > 2ATs\ ]$$

$$= P\ [V_{-+-+} + N < 0] + P[\ V_{-+-+} + N > 2ATs]$$

$$= P1 + P2$$

but $$V_{-+-+} = ATs\ (1 + \delta)$$

so

$$P1 = \frac{1}{2} erfc[(\sqrt{z})\ (1+\delta)] \text{ and}$$

$$P2 = \frac{1}{2} erfc[(\sqrt{z})\ (1-\delta)]$$

hence

$$P(E/0101) = \frac{1}{2}\ [erfc[(\sqrt{z})\ (1+\delta)] + erfc[(\sqrt{z})\ (1-\delta)]]$$

$$P(E/1101) = P\ [V_{++-+} + N < 0 \quad \text{or } V_{++-+} + N > 2ATs\ ]$$

$$= P\ [V_{++-+} + N < 0] + P[\ V_{++-+} + N > 2ATs]$$

$$= P1 + P2$$

but $V_{++-+} = ATs\ (1 + \delta + \delta(\frac{\tau_m}{Ts}))]$ so

$$P(E/1101) = \frac{1}{2}\ [erfc[(\sqrt{z})\ (1 + \delta + \delta(\frac{\tau_m}{Ts})]$$

$$+ \quad erfc[(\sqrt{z})\ (1 - \delta - \delta(\frac{\tau_m}{Ts})]]$$

$$P(E/1001) = P\ [V_{+--+} + N < 0 \quad \text{or } V_{+--+} + N > 2ATs\ ]$$

$$= P\ [V_{+--+} + N < 0] + P[\ V_{+--+} + N > 2ATs]$$

$$= P1 + P2$$

but $$V_{+--+} = ATs\ (1 + \delta - \delta(\frac{\tau_m}{Ts})$$

so

$$P(E/1001) = \frac{1}{2}\ [erfc[(\sqrt{z})\ (1 + \delta - \delta(\frac{\tau_m}{Ts})]$$

$$+\ erfc[(\sqrt{z})\ (1 - \delta + \delta(\frac{\tau_m}{Ts})]]$$

Finally

$$P(E/0001) = P\ [V_{---+} + N < 0\quad or\ V_{---+} + N > 2ATs\ ]$$

$$= P\ [V_{---+} + N < 0] + P[\ V_{---+} + N > 2ATs]$$

$$= P1 + P2$$

but $$V_{---+} = ATs\ (1 + \delta - 2\delta(\frac{\tau_m}{Ts})\ \text{so}$$

$$P(E/0001) = \frac{1}{2}\ [erfc[(\sqrt{z})\ (1 + \delta - 2\delta(\frac{\tau_m}{Ts})]$$

$$+\ erfc[(\sqrt{z})\ (1 - \delta + 2\delta(\frac{\tau_m}{Ts})]]$$

$$Pe1 = 2/16\ [P(E/0011) + P(E/1011) + P(E/0111) + P(E/1111)$$
$$+ P(E/0001) + P(E/0101) + P(E/1001) + P(E/1101)]$$

$$Pe1 = Pe2$$

$$Pc = (\ 1 - Pe1)(\ 1 - Pe2)$$

$$= 2\ Pe1 - Pe1^2$$

Generally, the M-QAM probability expression can be found by:

$$Pe1 = \frac{2}{M}\left[\frac{1}{2}\sum_{k=0}^{m}\left[\sum_{\substack{i=-(m-1)\\ i\neq 0}}^{(m-1)} erfc\ z'[1 + l_i\delta + l_i(1-k)\delta(\frac{\tau}{Ts})]\right.\right.$$

$$\left.\left.+ erfc\ z'[1 + l_m\delta - k\delta(\frac{\tau}{Ts})]\right]\right]$$

where $m = \sqrt{M} - 1$

$$z' = \sqrt{\frac{3z}{2(M-1)}}$$

z : the average symbol S/N, and $Ts = T \log_2(M)$

Finally, we evaluate the probability by

$$Pe \simeq 2Pe1 - Pe1^2$$

APPENDIX IV

APPENDIX IV-a

```
***********************************************************
*THE PROGRAM  ESTIMATES PE FOR COHERANT AND NON-COHERENT *
*BINARY MODULATIONS  WITH GAUSSIAN NOISE                 *
***********************************************************
C
C       USE OF QATR (XL,XU,EPS,NDIM,FCT,Y,IER,AUX)
C       FROM THE SSPSYS PACKAGE
C
C
C    THE S/N RATIO IS TAKEN AS A**2/ 2*ETA*BT
C
C
      DIMENSION   AUX(100)
      EXTERNAL F1
    1 FORMAT(1F6.3,6(1X,1E10.4))
      PI=3.141595
      NDIM=100
      WRITE(6,3)
    3 FORMAT(/)

*********************************************
* BINARY COHERENT DIGITAL MODULATION SCHEMES*
*********************************************
C
C ASK  MODULATION
C ---------------
C
      DO 10 I = 1,60,2
      EPS=0.0000001
      B1= 13
      Z = 10.*ALOG10(FLOAT(I))
      A1= SQRT( FLOAT(I)/4.)
C
C
      CALL QATR (A1,B1,EPS,NDIM,F1,Y,IER,AUX)
      YY= 2.* Y/ SQRT(PI)
```

```
      PE1= 0.5*YY
C
C  FSK  MODULATION
C  ---------------
C
      A1= SQRT( 0.61*FLOAT(I))
      CALL QATR (A1,B1,EPS,NDIM,F1,Y,IER,AUX)
      YY= 2.* Y/ SQRT(PI)
      PE2= 0.5*YY
C
C  PSK  MODULATION
C  ---------------
C
      A1= SQRT( FLOAT(I))
      CALL QATR (A1,B1,EPS,NDIM,F1,Y,IER,AUX)
      YY= 2.* Y/ SQRT(PI)
      PE3= 0.5*YY
 10   WRITE(6,1)Z,PE1,PE2,PE3
C
C**************************************************
C  BINARY NON COHERENT DIGITAL MODULATION SCHEMES*
C**************************************************
C
C     WRITE(6,3)
C     WRITE(6,2)
C
C  ASK  MODULATION
C  ---------------
C
      DO 11 I = 1,60,2
      Z = 10.*ALOG10(FLOAT(I))
      A1= FLOAT(I)
      CALL EXPP(A1/8.,Y1)
      CALL QATR (A1/4.,B1,EPS,NDIM,F1,Y2,IER,AUX)
C
C
      PE4= ( 0.5*Y2/SQRT(PI))+ Y1
C
C  FSK  MODULATION
C  ---------------
C
      CALL EXPP(A1/4.,Y)
      PE5=  Y
C
C  DPSK  MODULATION
C  ----------------
C
      CALL EXPP(A1,Y)
      PE6=  Y
 10   WRITE(6,1)Z,PE1,PE2,PE3,PE4,PE5,PE6
      STOP
```

```
      END
C
      FUNCTION F1(X)
      F1 = EXP(-X**2)
      RETURN
      END
C
      SUBROUTINE EXPP( X , Y)
      Y = 0.5*EXP(-X)
      RETURN
```

APPENDIX IV-b

```
*************************************************************
* PROGRAM TO ESTIMATE ASK, PSK WITH FLAT FADING AND AWGN*
*************************************************************

      DIMENSION   AUX(100)
      EXTERNAL F1 ,F2
      PI=3.141595
      A1= -PI
      B1= PI
      NDIM=100
C
C
C
      DO 10 II = 1,5
      BETA = 0.1*FLOAT(II-1)
      PRINT,' '
      WRITE(6,6)BETA
    6 FORMAT(' THE VALUE OF BETA  ', 1F10.5)
      PRINT,' '
      DO 10 KK = 1,60,2
      ZO = FLOAT(KK)
      EPS=0.0000001
      A  = 10.*ALOG10(ZO)
   CALL QATR (A1,B1,BETA,ZO,EPS,NDIM,F1,Y,IER,AUX)
   YY= (EXP(-ZO/4.0))/(2.00*PI*(SQRT(PI*ZO)))
   YA = YY * Y
C
C  PSK MODULATION
```

```
C  --------------
C
   CALL QATR (A1,B1,BETA,ZO,EPS,NDIM,F2,Y,IER,AUX)
   YY= (EXP(-ZO))/(4.00*PI*(SQRT(PI*ZO)))
   YB = YY * Y
 10   WRITE(6,1)A ,YA,YB
  1   FORMAT(1F10.5,2E17.7)
      STOP
      END
C
C
C
      FUNCTION F2(X,BETA,ZO)
      XX= (COS(X))
      XY =-2.0*ZO*XX*BETA
      F11 = EXP(XY)
      F22 = (1.0 - BETA* XX)
      F2 = F11*F22
      RETURN
      END
C
C
C
      FUNCTION F1(X,BETA,ZO)
      XX= (COS(X))
      XY =-0.5*ZO*XX*BETA
      F11 = EXP(XY)
      F22 = (1.0 - BETA* XX)
      F1 = F11*F22
      RETURN
      END
```

APPENDIX IV-c

```
C*******************************************************
C PROGRAM TO ESTIMATE 4 AND 8-PSK WITH FLAT FADING AND  *
C AWGN                                                  *
C*******************************************************
    DIMENSION   AUX(100) ,Y1(100),Y2(100), Y3(100),Y4(100)
                EXTERNAL F1
                PI=3.141595
```

```
      A1= -PI
      B1= PI
      NDIM=100
C
      AM = 4.0
      PRINT,' THE VALUE OF M IS ',AM
C
C
C
C     DO 10 II = 1,4
      BETA = 0.0
C     PRINT,'  '
      DO 10 KK =19,69,2
      ZO = FLOAT(KK)
      EPS=0.0000001
      A  = 10.*ALOG10(ZO)
CALL QATR (A1,B1,BETA,ZO,AM,EPS,NDIM,F1,Y,IER,AUX)
 10   Y1(KK) =  Y
      BETA = 0.2
C     PRINT,'  '
      DO 12 KK =19,69,2
      ZO = FLOAT(KK)
      EPS=0.0000001
      A  = 10.*ALOG10(ZO)
CALL QATR (A1,B1,BETA,ZO,AM,EPS,NDIM,F1,Y,IER,AUX)
      Y2(KK) =  Y
 12   WRITE(6,1)A ,Y1(KK),Y2(KK)
C
      AM = 8.0
      PRINT,' THE VALUE OF M IS ',AM
C
C
C
C     DO 10 II = 1,4
      BETA = 0.0
C     PRINT,'  '
      DO 13 KK =19,69,2
      ZO = FLOAT(KK)
      EPS=0.0000001
      A  = 10.*ALOG10(ZO)
CALL QATR (A1,B1,BETA,ZO,AM,EPS,NDIM,F1,Y,IER,AUX)
 13   Y3(KK) =  Y
      BETA = 0.2
C     PRINT,'THE VALUE OF  BETA',BETA
C     PRINT,' '
C     PRINT,'    THE S/N IN DB','     PE VALUE '
C     PRINT,'  '
      DO 14 KK =19,69,2
      ZO = FLOAT(KK)
      EPS=0.0000001
      A  = 10.*ALOG10(ZO)
```

```
      CALL QATR (A1,B1,BETA,ZO,AM,EPS,NDIM,F1,Y,IER,AUX)
            Y4(KK) =  Y
      C     DO 11 I =19,69,2
       14   WRITE(6,1)A ,Y3(KK),Y4(KK)
        1   FORMAT(5E15.5)
            STOP
            END
      C
      C
            FUNCTION F1(X,BETA,ZO,AM)
            PI = 3.141595
            XX= (COS(X))
            F  = (1.0D0 + BETA* XX)**2
            F1 = EXP( -F*ZO*((SIN(PI/AM))**2))
            RETURN
            END
```

APPENDIX IV-d

```
C***********************************************************
C                                                          *
C  ZERO FORCING AND MEAN SQUARE ERROR EQUALIZERS           *
C                                                          *
C  PERFORMANCES ANALYSIS WITH THE THREE MODELS, THE        *
C                                                          *
C  TWO RAY , THE THREE RAY AND THE POLYNIMIAL MODELS       *
C                                                          *
C***********************************************************
C
C  THE FOLLOWING SUBROUTINES  ARE TAKEN FROM  THE SSP
C  LIBRARY: GELE , MTRA , MPRD , MINV
C
C
   DIMENSION A(200) ,B(200),BC(200),X0(200),W(1000),YK(200)
   DIMENSION S(200) ,HN(200),XV(200,200),X(200),XO(200)
   DIMENSION TN(200) ,Y(200,200),Z(200,200),EH(200),ZZ(400)
   DIMENSION C(200) ,YC(200,200),XO2(200),XOI(200),CC(200)
   DIMENSION YT(400),RR(400),MKK(200),KM(200),YY(400)
   DIMENSION XT(200),ZT(200),MX(200),E(200),XI(200),CM(200)
         C
         C
```

```
C
      II = 16
      K  = 16
C
C
C THE STREAM OF DATA
C ------------------
      PRINT ,' THE STREAM OF DATA'
      WRITE(6,31)
      DO 12 I=1,II
      X(I) = (-1.0)**(I)
 12   WRITE(6,77)  I , X(I)
C  THE PULSE SHAPE OF THE STREAM
   77 FORMAT( 6X, I3 , 6X,F10.5)
C
C CHANNEL IMPULSE RESPONSEs
C -------------------------
C TWO-RAY MODEL
C -------------
C
      DO 1100 LC=1,20
 1100 CALL  2RAYM(LC,DT,NT,FK)
      DO 100 I=1,N
 100  FKA(I) = CABS(FK(I))
C
C
      CALL IFFT(N,DT,FK,FT)
      DO 105 IF=1,N
      N1 =IF-1
      HN(IF) = CABS(FT(IF))
 105  PRINT, N1 ,FKA(IF) ,HN(IF)
C
C  WE REARRANGE THE H RESPONSE
C
      DO 15 I =1,K
   15 HN1(I) = HN(I)
      CALL ARRNG(K ,HN1,HN)
C
C
C THREE-RAY MODEL
C ---------------
C
      DO 1101 LC=1,300
1101 CALL  3RAYM(LC,DT,NT,FK)
      DO 101 I=1,N
101   FKA(I) = CABS(FK(I))
C
C
      CALL IFFT(N,DT,FK,FT)
      DO 106 IF=1,N
      N1 =IF-1
```

```
      HN(IF) = CABS(FT(IF))
 106  PRINT, N1 ,FKA(IF) ,HN(IF)
C
C  WE REARRANGE THE H RESPONSE
C
      DO 16 I =1,K
   16 HN1(I) = HN(I)
      CALL ARRNG(K ,HN1,HN)
C
C
C POLYNOMIAL MODEL
C -----------------
C
      DO 1100 LC=1,9
 1102 CALL  POLYM(LC,DT,NT,FK)
      DO 102 I=1,N
 102  FKA(I) = CABS(FK(I))
C
C
      CALL IFFT(N,DT,FK,FT)
      DO 107 IF=1,N
      N1 =IF-1
      HN(IF) = CABS(FT(IF))
 107  PRINT, N1 ,FKA(IF) ,HN(IF)
C
C  WE REARRANGE THE H RESPONSE
C
      DO 17 I =1,K
   17 HN1(I) = HN(I)
C
C
      CALL ARRNG(K ,HN1,HN)
C
C
      WRITE(6,31)
  31  FORMAT(//)
      WRITE(6,31)
      DO 11 I =1,K
  11  WRITE(6,151) I,HN(I)
      WRITE(6,31)
 151  FORMAT(2X,'HN(',I3,') = ',1F10.7)
 1    FORMAT(I3,4X,1F10.7)
      CALL AMAXX(II,HN,AMAX,AMIN)
      CALL PEAK ( HN,II,AMAX,DP)
      CALL SQUARD(HN,II,AMAX,DM)
      CALL ERROR (HN,II,AMAX,ER)
      WRITE(6,31)
      LL = K+II-1
C
C THE OUTPUT OF A STREAM X(T)
C --------------------------
```

```
C
      PRINT,'THE OUTPUT TO THE  STREAM X(T)'
      WRITE(6,31)
      CALL OUTPUT( K,II,X,HN,XV)
      CALL XOUTT(K,II,XV,XO)
      DO 59 I =1,LL
 59   WRITE(6,1) I, XO(I)
      WRITE(6,31)
C
C
C
      DO 1000 M = 3,11,2
C     M = 5
      WRITE(6,31)
      WRITE( 6,90) M
  90  FORMAT( 20X,'EQUALIZER DESIGN FOR M=',I5)
      WRITE(6,31)
      PRINT ,' ZERO FORCING EQUALIZER DESIGN '
C
C
C  GENERATION OF MATRIX Y OF DIMENSION M*(M+II-1)
C  ---------------------------------------------
      MM =M+II-1
      CALL AMATY(M,II,HN,Y)
C
C
C TRANCATED Z MATRIX GENERATION
C ------------------------------
C JJ IS THE HN MAX COMPONENT TERM
      DO 39 I = 1,II
   39 IF ( AMAX .EQ.HN(I)) JJ=I
      PRINT , 'JJ = ', JJ
      LJ = MM -(II-JJ)
      JL = JJ-1
      DO 23 I = 1,M
      DO 23 J = JJ,LJ
      KA = J-JL
 23   Z(I,KA) = Y(I,J)
C
C EH MATRIX GENERATION
C --------------------
      PRINT,' EH MATRIX GENERATION '
      CALL EHMAT(M,EH)
      WRITE(6,47)(EH(I),I=1,M)
  47  FORMAT(1E17.7)
C  THE COEFFICIENTS CALCULATIONS
C
C  CONVERSION OF MATRIX Z TO AN ARRAY
C  ---------------------------------
      CALL ARRAY(M,M,Z,ZZ)
C
```

```
      DO 53 I =1,M
   53 C(I) = EH(I)
      MN = M**2
      EPS = 0.000001
      N=1
      CALL GELG(C,ZZ,M,N,EPS,IER)
      PRINT,'THE VALUES OF THE COEFFICIENTS'
      WRITE(6,31)
      DO 57 I =1,M
      MK = M+1-I
      SUM = C(I)
   57 CC(MK) = SUM
      DO 58 I = 1,M
   58 WRITE(6,48) I,CC(I)
      WRITE(6,31)
   48 FORMAT(2X,'C (',I3,')=',1F10.7)
C
C THE OUTPUT OF THE EQUALIZER
C
      CALL OUTPUT(M,II,CC,HN,XV)
      LL = M+II-1
C
C THE ONE DIMENSIONAL OUTPUT
C
      CALL XOUTT(M,II,XV,XO)
      PRINT,'THE OUTPUT OF THE EQUALIZER'
      WRITE(6,31)
      DO 55 L=1,LL
   55 WRITE(6,1) L,XO(L)
      MM = M+II-1
      CALL EHMAT(MM,EH)
      CALL AMAXX(LL,XO,AMAX,AMIN)
      CALL PEAK(XO,LL,AMAX,DP)
      CALL SQUARD(XO,LL,AMAX,DM)
      CALL ERROR( XO,LL,AMAX,ER)
C
C THE STREAM OUTPUT OF THE EQUALIZER
C
      PRINT,'THE EQUALIZED INPUT STREAM '
      CALL OUTPUT(K,LL,X,XO,XV)
      CALL XOUTT(K,LL,XV,XO2)
      IK = K+LL -1
      DO 56 L=1,IK
   56 WRITE(6,1) L,XO2(L)
C
C*************************************************
C EQUALIZER WITH MINIMUM MEAN SQUARE  ERROR       *
C*************************************************
C
      WRITE(6,31)
C
```

```
C
      CALL AMATY(M,II,HN,Y)
C
C
C  Y MATRIX TRANSPOSE
C  CONVERSION OF Y MATRIX TO AN ARRAY
      CALL ARRAY (M,MM,Y,ZZ)
C
C
      CALL MTRA(ZZ,YT,M,MM,0)
      CALL MPRD(ZZ,YT,RR,M,MM,0,0,M)
      CALL MINV(RR,M,D,KM,MKK)
      CALL MPRD(YT,RR,YY,MM,M,0,0,M)
      CALL MPRD(EH,YY,CM,1,MM,0,0,M)
      WRITE(6,31)
      PRINT,'THE VALUES OF THE COFFICIENTS'
      DO 60 I=1,M
  60  WRITE(6,48) I,CM(I)
      WRITE(6,31)
C
      PRINT,'THE OUTPUT OF THE EQUALIZER'
      CALL OUTPUT(M,II,CM,HN,XV)
      CALL XOUTT(M,II,XV,XO)
      DO 64 I=1,MM
 64   WRITE(6,1) I,XO(I)
      WRITE(6,31)
      CALL AMAXX(MM,XO,AMAX,AMIN)
      CALL PEAK(XO,MM,AMAX,DP)
      CALL SQUARD(XO,MM,AMAX,DM)
      CALL ERROR( XO,MM,AMAX,ER)
      PRINT,'THE EQUALIZED  INPUT STREAM '
      CALL OUTPUT(K,LL,X,XO,XV)
      CALL XOUTT(K,LL,XV,XO2)
      IK = K+LL-1
      DO 66 L=1,IK
 66   WRITE(6,1)L,XO2(L)
 1000 CONTINUE
 1001 CONTINUE
 102  CONTINUE
      STOP
      END
C
C SUBROUTINE  FOR FINDING THE MAX AND  THE MIN
C ---------------------------------------------
C
      SUBROUTINE AMAXX(M,A,AMAX,AMIN)
      DIMENSION A(200),B(200),BC(200)
      DO 70 I=1,M
      BC(I) = A(I)
  70  B(I) = ABS(A(I))
      MZ = M-1
```

```
      DO 75 I=1,MZ
      DO 75 J=1,MZ
      LK = J+1
      IF(B(J).LE.B(LK)) GO TO 75
      DX = B(J)
      DY = BC(J)
      B(J) = B(LK)
      BC(J) = BC(LK)
      B(LK) = DX
      BC(LK)=DY
 75   CONTINUE
      AMAX = BC(M)
      AMIN = BC(1)
      RETURN
      END
C
C
C  SUBROUTINE FOR PEAK  DISTORTION
C  -------------------------------
C
      SUBROUTINE PEAK(HN,II,AMAX,DP)
      DIMENSION HN(200)
      SUM = 0.0
      DO 20 I=1,II
 20   SUM = ABS(HN(I)) + SUM
      DP = ( SUM/AMAX)  -1.
      PRINT,'THE PEAK DISTORTION IS',DP
      RETURN
      END
C
C  SUBROUTINE FOR MEAN SQUARE DISTORTION
C  -------------------------------------
C
      SUBROUTINE SQUARD(HN,II,AMAX,DM)
      DIMENSION  HN(200)
      SUM = 0.0
      DO 11 I=1,II
 11   SUM = HN(I)**2  + SUM
      DM = (SUM/(AMAX**2)) -1.0
      PRINT,' THE MEAN SQUARE DISTORTION IS',DM
      RETURN
      END
C
C  SUBROUTINE FOR  MEAN SQUARE ERROR
C  ---------------------------------
C
      SUBROUTINE ERROR(HN,II,AMAX,ER)
      DIMENSION  HN(200)
      SUM = 0.0
      DO 10 I=1,II
 10   SUM = HN(I)**2  +  SUM
```

```
      ER = (SUM/(AMAX**2))  -1.0
      ER = ER + ( AMAX -1.0)**2
      PRINT,' THE MEAN SQUARE ERROR IS',ER
      RETURN
      END
C
C  SUBROUTINE FOR THE OUTPUT TO A STREAM X(T)
C  -----------------------------------------
C
      SUBROUTINE OUTPUT( K,II,X,HN,XV)
      DIMENSION X(200),HN(200),XV(200,200)
      LL = K+II-1
      DO 12 I=1,K
      DO 12 J=1,LL
      IF((I.GT.J).OR.(J.GT.(II+I-1))) XV(I,J)=0.0
   12 CONTINUE
      DO 13 I=1,K
      DO 13 J=1,LL
      IF((J.GE.I).AND.(J.LE.(II+I-1))) XV(I,J)
      * =X(I)*HN(J-I+1)
   13 CONTINUE
      RETURN
      END
C
C THE ONE SUBROUTINE XOUTPUT
C -------------------------
C
      SUBROUTINE XOUTT(K,II,XV,XO)
      DIMENSION XV(200,200),XO(200)
      LL = K+II-1
      DO 18 J=1,LL
      SUM = 0.0
      DO 19 I=1,K
  19  SUM = XV(I,J) + SUM
  18  XO(J) = SUM
      RETURN
      END
C
C GENERATION OF MATRIX Y OF DIMENSION M*(M+II-1)
C ---------------------------------------------
C
      SUBROUTINE AMATY(M,II,HN,Y)
      DIMENSION HN(200),Y(200,200)
      MM = M+II-1
      DO 21 I=1,M
      DO 21 J=1,MM
      IF((I.GT.J).OR.(J.GT.(II+I-1))) Y(I,J)=0.0
   21 CONTINUE
      DO 22 I=1,M
      DO 22 J=1,MM
      IF((J.GE.I).AND.(J.LE.(II+I-1))) Y(I,J)
```

```
     * = HN(J-I+1)
 22   CONTINUE
      RETURN
      END
C
C EH MATRIX GENERATION
C --------------------
C
      SUBROUTINE EHMAT(M,EH)
      DIMENSION EH(200)
      DO 24 I = 1,M
      MI = M
      MI = ((MI-1)/2) +1
 24   IF(I.EQ.MI) EH(I) =1.0
      DO 25 I=1,M
      MI = M
      MI =((MI-1)/2)+1
 25   IF(I.NE.MI) EH(I)=0.0
      RETURN
      END
C
C SUBROUTINE TO CONVERT MATRIX TO AN ARRAY FOR SSP
C ------------------------------------------------
C
      SUBROUTINE ARRAY(M,N,Z,ZZ)
      DIMENSION ZZ(400),Z(200,200)
      DO 28 J=1,N
      DO 28 I=1,M
      KK = (J-1)*(M-1)
      KI =(I+J-1) + KK
 28   ZZ(KI) = Z(I,J)
      RETURN
      END
C
C
C  SUBROUTINE FOR BB PULSE
C  -----------------------
C
      SUBROUTINE PULSE( KK ,RO,KTAU,BETA,X)
      PI = 3.141596
      AK = KK
      AKTAU = KTAU
      SK = ( AK - AKTAU/10.0)
      IF (SK.EQ.0.0) GO TO 5
      X =(SIN(2.0*PI*SK))/(2.0*PI*SK)
      GO TO 6
  5   X =1.0
  6   X = X*(COS(2.0*PI*RO*SK))
     */(1.0-(4.0*RO*SK)**2)
      X = BETA * X
      RETURN
```

```
      END
C
C
C  TWO-RAY MODEL
C  -----------
C
      SUBROUTINE 2RAYM(LC,DT,NT,FK)
      TAU = (LC)*DT
      LK = LC-1
      PRINT,' TAU = OF DT',LK
      N =NT/DT
      WRITE(6,6)
      DO 100 I=1,N
      N1 =I-1
      SF= FLOAT(N1)
      DF = 1/(N*DT)
      DF = SF*DF
      AA = CMPLX(0.0,-2.0*PI*DF*TAU)
  100 FK(I) = 1.0+  BETA*CEXP( AA)
      RETURN
      END
C
C
C POLYNOMIAL MODEL
C ----------------
C
      SUBROUTINE POLYM(LC,DT,NT,FK)
      AO = 4.0-(LC-1)
      AO = 1.0
      AO = (AO*(6.562)+(-21.39))/20.0
      AO = 10.0**AO
      PRINT,AO
      A1 =  0.01*AO
      B1 =  0.01*AO
      N =NT/DT
      PRINT,N
      WRITE(6,6)
      DO 100 I=1,N
      N1 =I-1
      SF= FLOAT(N1)
      DF = 1/(N*DT)
      DW = SF*DF*2.0*PI
      AA = CMPLX(AO-DW*B1, DW*A1)
 100  FK(I) = AA
      RETURN
      END
C
C
C THREE-RAY MODEL
C ---------------
C
```

```
      SUBROUTINE 3RAYM(LC,DT,NT,FK)
      TAU = (LC-1)*DT*6.61E-09
      LK = LC-1
      PRINT,' TAU = OF DT',LK
      N =NT/DT
      WRITE(6,6)
      DO 100 I=1,N
      N1 =I-1
      SF= FLOAT(N1)
      DF = 1/(N*DT)
      FFO= SF*DF*10.0E06
      AA = CMPLX(0.0,-2.0*PI*FFO*TAU)
 100  FK(I) = 1.0+  BETA*CEXP( AA)
      RETURN
      END
C
C  SUBROUTINE FOR FAST-FOURRIER TRANSFORM
C  -------------------------------------
c

      SUBROUTINE FFT(N,DT,FT, FK)
      INTEGER MM
      REAL X
      COMPLEX U ,W,X1,FK(1500) ,FT(1500)
C BIT REVERSAL OPERATION
      PRINT,N
      X = ALOG(FLOAT(N))/ALOG(2.0)
      F = 0.1
      MM = INT(X+F)
      N2 = N/2
      N3 = N -1
      J = 1
      DO 400 I = 1,N3
      IF(I.GE.J) GO TO 200
      X1 = FT(J)
      FT(J) = FT(I)
      FT(I) = X1
 200  K = N2
 300  IF(K.GE.J) GO TO 400
      J = J-K
      K = K/2
      GO TO 300
 400  J = J + K
      PI = 22./7.
      DO 20 L = 1,MM
      N4 = 2**L
      N5 = N4/2
      U = (1.0 ,0.0)
      Z1 = 2.*PI/FLOAT(N4)
      W = CMPLX(COS(Z1),-SIN(Z1))
      DO 20 J=1,N5
```

```
      DO 10 I = J,N,N4
      IP = I + N5
      X1 = FT(IP)*U
      FT(IP) = FT(I)-X1
 10   FT(I) = FT(I) + X1
 20   U = U*W
      DO 40 I= 1,N
      FK(I) = CABS(FT(I))*DT
 40   CONTINUE
      RETURN
      END
C
C
C
C SUBROUTINE FOR THE INVERSE FAST FORRIER TRANSFORM
C ---------------------------------------------------
C
      SUBROUTINE IFFT(N,DT,FW, FT)
      INTEGER MM
      REAL X
      COMPLEX U ,W,X1,FW(1500) ,FT(1500)

C BIT REVERSAL OPERATION

      X = ALOG(FLOAT(N))/ALOG(2.0)
      F = 0.1
      MM = INT(X+F)
      N2 = N/2
      N3 = N -1
      J = 1
      DO 400 I = 1,N3
      IF(I.GE.J) GO TO 200
      X1 = FW(J)
      FW(J) = FW(I)
      FW(I) = X1
 200  K = N2
 300  IF(K.GE.J) GO TO 400
      J = J-K
      K = K/2
      GO TO 300
 400  J = J + K
      PI = 22./7.
      DO 20 L = 1,MM
      N4 = 2**L
      N5 = N4/2
      U = (1.0 ,0.0)
      Z1 = 2.*PI/FLOAT(N4)
      W = CMPLX(COS(Z1), SIN(Z1))
      DO 20 J=1,N5
      DO 10 I = J,N,N4
      IP = I + N5
```

```
      X1 = FW(IP)*U
      FW(IP) = FW(I)-X1
 10   FW(I) = FW(I) + X1
 20   U = U*W
      DO 40 I= 1,N
      FT(I) = CABS(FW(I))
      FT(I) = FT(I)/(DT*N)
 40   CONTINUE
      RETURN
      END
C
C  SUBROUTINE FOR ARRANGING THE IMPULSE RESPONSE
C  ------------------------------------------------
C  TO PLACE THE HIGHEST TERM IN THE MIDDLE
C  ------------------------------------------
C
      SUBROUTINE ARRNG(K,HN1,HN)
      DIMENSION HN(200) ,HN1(200)
      KK = K/2
      DO 11 I =1,K
      IF(I.LE.(KK+1)) GO TO 5
      I1 = I-(KK+1)
      HN( I1) = HN1(I)
      GO TO 11
    5 I2 = I+ KK -1
      HN(I2)=HN1(I)
   11 CONTINUE
      RETURN
      END
```

APPENDIX IV-e

```
* ****************************************************
*                                                    *
*   THE PERFORMANCE ANALYSIS OF THE EQUALIZERS       *
*   FOR THE TWO-RAY MODEL                            *
*   BETA = 0.5 AND TAU/T = 0.31                      *
*                                                    *
******************************************************
```

THE INPUT STREAM OF DATA

```
-----------------------

X(  1) =   -1.00000
X(  2) =    1.00000
X(  3) =   -1.00000
X(  4) =    1.00000
X(  5) =   -1.00000
X(  6) =    1.00000
X(  7) =   -1.00000
X(  8) =    1.00000
X(  9) =   -1.00000
X( 10) =    1.00000
X( 11) =   -1.00000
X( 12) =    1.00000
X( 13) =   -1.00000
X( 14) =    1.00000
X( 15) =   -1.00000
X( 16) =    1.00000

THE SAMPLED INPULSE RESPONSE
----------------------------

H(  1) =  0.0005947
H(  2) =  0.0006332
H(  3) =  0.0004552
H(  4) =  0.0005373
H(  5) =  0.0000028
H(  6) =  0.0005395
H(  7) =  0.0004556
H(  8) =  2.6670190
H(  9) =  0.0005945
H( 10) =  0.0009455
H( 11) =  0.0010996
H( 12) =  0.0026876
H( 13) =  1.3333230
H( 14) =  0.0026973
H( 15) =  0.0011002
H( 16) =  0.0009459


THE PEAK DISTORTION                  0.5049114
THE MEAN SQUARE DISTORTION           0.2499323
THE MEAN SQUARE ERROR                3.0288860

THE RESULTED STREAM OF DATA
---------------------------

Y(  1) =    -0.0006
Y(  2) =    -0.0000
Y(  3) =    -0.0004
```

```
 Y(  4) =     -0.0001
 Y(  5) =      0.0001
 Y(  6) =     -0.0007
 Y(  7) =      0.0002
 Y(  8) =     -2.6672
 Y(  9) =      2.6666
 Y( 10) =     -2.6676
 Y( 11) =      2.6665
 Y( 12) =     -2.6692
 Y( 13) =      1.3358
 Y( 14) =     -1.3385
 Y( 15) =      1.3374
 Y( 16) =     -1.3384
 Y( 17) =      1.3390
 Y( 18) =     -1.3383
 Y( 19) =      1.3388
 Y( 20) =     -1.3383
 Y( 21) =      1.3383
 Y( 22) =     -1.3377
 Y( 23) =      1.3382
 Y( 24) =      1.3288
 Y( 25) =     -1.3282
 Y( 26) =      1.3292
 Y( 27) =     -1.3281
 Y( 28) =      1.3308
 Y( 29) =      0.0025
 Y( 30) =      0.0002
 Y( 31) =      0.0009

      EQUALIZER DESIGN FOR M=    5
****************************************

ZERO FORCING EQUALIZER DESIGN
*****************************


THE VALUES OF THE COEFFICIENTS
------------------------------


 C (  1)=-0.0000758
 C (  2)=-0.0000640
 C (  3)= 0.3749506
 C (  4)=-0.0000835
 C (  5)=-0.0001328


THE PEAK DISTORTION                 0.5034809
THE MEAN SQUARE DISTORTION          0.2499294
THE MEAN SQUARE ERROR               0.2499294
```

THE EQUALIZED STREAM OF DATA

```
Y(  1) =    0.0000
Y(  2) =    0.0000
Y(  3) =   -0.0002
Y(  4) =   -0.0000
Y(  5) =   -0.0002
Y(  6) =   -0.0000
Y(  7) =    0.0000
Y(  8) =   -0.0000
Y(  9) =    0.0000
Y( 10) =   -1.0000
Y( 11) =    1.0000
Y( 12) =   -1.0000
Y( 13) =    0.9997
Y( 14) =   -1.0007
Y( 15) =    0.5007
Y( 16) =   -0.5016
Y( 17) =    0.5014
Y( 18) =   -0.5017
Y( 19) =    0.5020
Y( 20) =   -0.5017
Y( 21) =    0.5019
Y( 22) =   -0.5017
Y( 23) =    0.5017
Y( 24) =   -0.5017
Y( 25) =    0.5017
Y( 26) =    0.4983
Y( 27) =   -0.4983
Y( 28) =    0.4983
Y( 29) =   -0.4980
Y( 30) =    0.4989
Y( 31) =    0.0010
Y( 32) =   -0.0001
Y( 33) =    0.0004
Y( 34) =   -0.0000
Y( 35) =   -0.0000
```

EQUALIZER WITH MINIMUM MEAN SQUARE ERROR
****

THE VALUES OF THE COFFICIENTS

```
C (   1)=-0.0001261
C (   2)=-0.0002697
C (   3)= 0.2999771
C (   4)=-0.0002853
C (   5)=-0.0001718
```

```
THE PEAK DISTORTION                  0.5044222
THE MEAN SQUARE DISTORTION           0.2499294
THE MEAN SQUARE ERROR                0.2899117
```

```
THE EQUALIZED  INPUT STREAM
----------------------------

 Y(  1) =     0.0000
 Y(  2) =     0.0000
 Y(  3) =    -0.0002
 Y(  4) =    -0.0000
 Y(  5) =    -0.0001
 Y(  6) =    -0.0000
 Y(  7) =     0.0000
 Y(  8) =     0.0001
 Y(  9) =     0.0004
 Y( 10) =    -0.8005
 Y( 11) =     0.8011
 Y( 12) =    -0.8009
 Y( 13) =     0.8007
 Y( 14) =    -0.8012
 Y( 15) =     0.4012
 Y( 16) =    -0.4016
 Y( 17) =     0.4015
 Y( 18) =    -0.4018
 Y( 19) =     0.4020
 Y( 20) =    -0.4018
 Y( 21) =     0.4020
 Y( 22) =    -0.4018
 Y( 23) =     0.4018
 Y( 24) =    -0.4020
 Y( 25) =     0.4014
 Y( 26) =     0.3987
 Y( 27) =    -0.3992
 Y( 28) =     0.3991
 Y( 29) =    -0.3989
 Y( 30) =     0.3994
 Y( 31) =     0.0006
 Y( 32) =    -0.0002
 Y( 33) =     0.0003
 Y( 34) =    -0.0000
 Y( 35) =    -0.0000
```

APPENDIX IV-f

```
*****************************************************
*                                                   *
*   THE PERFORMANCE ANALYSIS OF THE EQUALIZERS      *
*   FOR THE THREE-RAY MODEL                         *
*   BETA = 0.5 AND TAU/T = 0.31                     *
*   TAU  = 6.31 NS  AND FO IN MHZ                   *
*****************************************************
```

THE SAMPLED INPULSE RESPONSE

```
H(  1) =    0.00937
H(  2) =    0.00809
H(  3) =    0.00715
H(  4) =    0.00683
H(  5) =    0.00617
H(  6) =    0.00684
H(  7) =    0.00713
H(  8) =    2.67162
H(  9) =    0.00928
H( 10) =    0.01198
H( 11) =    0.01703
H( 12) =    0.03349
H( 13) =    1.33191
H( 14) =    0.03522
H( 15) =    0.01743
H( 16) =    0.01215
```

```
THE PEAK DISTORTION                 0.5689678
THE MEAN SQUARE DISTORTION          0.2490625
THE MEAN SQUARE ERROR               3.0433760
```

THE RESULTED STREAM OF DATA

```
Y(  1) =    -0.0094
Y(  2) =     0.0013
Y(  3) =    -0.0084
Y(  4) =     0.0016
```

```
Y(  5) =    -0.0078
Y(  6) =     0.0009
Y(  7) =    -0.0081
Y(  8) =    -2.6636
Y(  9) =     2.6543
Y( 10) =    -2.6663
Y( 11) =     2.6492
Y( 12) =    -2.6827
Y( 13) =     1.3508
Y( 14) =    -1.3860
Y( 15) =     1.3686
Y( 16) =    -1.3807
Y( 17) =     1.3901
Y( 18) =    -1.3820
Y( 19) =     1.3892
Y( 20) =    -1.3823
Y( 21) =     1.3885
Y( 22) =    -1.3817
Y( 23) =     1.3888
Y( 24) =     1.2828
Y( 25) =    -1.2735
Y( 26) =     1.2855
Y( 27) =    -1.2685
Y( 28) =     1.3020
Y( 29) =     0.0299
Y( 30) =     0.0053
Y( 31) =     0.0122


      EQUALIZER DESIGN FOR M=    5
****************************************

ZERO FORCING EQUALIZER DESIGN
*****************************


THE VALUES OF THE COEFFICIENTS
------------------------------

 C (   1)=  -0.00095
 C (   2)=  -0.00099
 C (   3)=   0.37432
 C (   4)=  -0.00129
 C (   5)=  -0.00166


THE PEAK DISTORTION                 0.5487661
THE MEAN SQUARE DISTORTION          0.2487946
THE MEAN SQUARE ERROR               0.2487946
```

THE EQUALIZED INPUT STREAM

```
Y(  1) =     0.0000
Y(  2) =     0.0000
Y(  3) =    -0.0035
Y(  4) =     0.0005
Y(  5) =    -0.0031
Y(  6) =     0.0006
Y(  7) =    -0.0029
Y(  8) =     0.0029
Y(  9) =    -0.0029
Y( 10) =    -0.9971
Y( 11) =     0.9971
Y( 12) =    -0.9971
Y( 13) =     0.9921
Y( 14) =    -1.0032
Y( 15) =     0.5048
Y( 16) =    -0.5162
Y( 17) =     0.5119
Y( 18) =    -0.5164
Y( 19) =     0.5199
Y( 20) =    -0.5169
Y( 21) =     0.5195
Y( 22) =    -0.5170
Y( 23) =     0.5193
Y( 24) =    -0.5193
Y( 25) =     0.5193
Y( 26) =     0.4807
Y( 27) =    -0.4807
Y( 28) =     0.4807
Y( 29) =    -0.4757
Y( 30) =     0.4868
Y( 31) =     0.0116
Y( 32) =    -0.0002
Y( 33) =     0.0045
Y( 34) =    -0.0000
Y( 35) =    -0.0000
```

EQUALIZER WITH MINIMUM MEAN SQUARE ERROR
****

THE VALUES OF THE COFFICIENTS

```
C (   1)=  -0.00185
C (   2)=  -0.00348
```

```
 C (   3)=   0.29983
 C (   4)=  -0.00372
 C (   5)=  -0.00243

THE PEAK DISTORTION                    0.5606728
THE MEAN SQUARE DISTORTION             0.2485552
THE MEAN SQUARE ERROR                  0.2881861

THE EQUALIZED  INPUT STREAM
---------------------------

 Y(   1) =     0.0000
 Y(   2) =     0.0000
 Y(   3) =    -0.0028
 Y(   4) =     0.0004
 Y(   5) =    -0.0025
 Y(   6) =     0.0005
 Y(   7) =    -0.0023
 Y(   8) =     0.0053
 Y(   9) =     0.0020
 Y(  10) =    -0.8029
 Y(  11) =     0.8102
 Y(  12) =    -0.8071
 Y(  13) =     0.8046
 Y(  14) =    -0.8099
 Y(  15) =     0.4109
 Y(  16) =    -0.4163
 Y(  17) =     0.4145
 Y(  18) =    -0.4180
 Y(  19) =     0.4209
 Y(  20) =    -0.4185
 Y(  21) =     0.4205
 Y(  22) =    -0.4186
 Y(  23) =     0.4203
 Y(  24) =    -0.4233
 Y(  25) =     0.4161
 Y(  26) =     0.3849
 Y(  27) =    -0.3921
 Y(  28) =     0.3891
 Y(  29) =    -0.3866
 Y(  30) =     0.3919
 Y(  31) =     0.0072
 Y(  32) =    -0.0017
 Y(  33) =     0.0036
 Y(  34) =    -0.0001
 Y(  35) =    -0.0000
```

APPENDIX IV-g

```
*****************************************************
*                                                   *
*   THE PERFORMANCE ANALYSIS OF THE EQUALIZERS      *
*   FOR THE TWO-RAY MODEL                           *
*   AO = 0.1813, A1 = 0.001*AO AND B1 = 0.01*AO     *
*                                                   *
*****************************************************
```

```
THE SAMPLED IMPULSE  RESPONSE
-----------------------------

H(  1) =  0.0036533
H(  2) =  0.0038786
H(  3) =  0.0043073
H(  4) =  0.0050688
H(  5) =  0.0064515
H(  6) =  0.0093678
H(  7) =  0.0183777
H(  8) =  0.4473045
H(  9) =  0.0183590
H( 10) =  0.0093595
H( 11) =  0.0064430
H( 12) =  0.0050656
H( 13) =  0.0043071
H( 14) =  0.0038751
H( 15) =  0.0036450
H( 16) =  0.0035831


THE PEAK DISTORTION                   0.2363977
THE MEAN SQUARE DISTORTION            0.0054522
THE MEAN SQUARE ERROR                 0.3109244


THE RESULT STREAM OF DATA
-------------------------


Y(  1) =   -0.00365
Y(  2) =   -0.00023
Y(  3) =   -0.00408
Y(  4) =   -0.00099
Y(  5) =   -0.00546
```

```
Y(  6) =   -0.00390
Y(  7) =   -0.01447
Y(  8) =   -0.43283
Y(  9) =    0.41447
Y( 10) =   -0.42383
Y( 11) =    0.41739
Y( 12) =   -0.42245
Y( 13) =    0.41815
Y( 14) =   -0.42202
Y( 15) =    0.41838
Y( 16) =   -0.42196
Y( 17) =    0.42561
Y( 18) =   -0.42173
Y( 19) =    0.42604
Y( 20) =   -0.42097
Y( 21) =    0.42742
Y( 22) =   -0.41806
Y( 23) =    0.43643
Y( 24) =    0.01087
Y( 25) =    0.00749
Y( 26) =    0.00187
Y( 27) =    0.00457
Y( 28) =    0.00049
Y( 29) =    0.00381
Y( 30) =    0.00006
Y( 31) =    0.00358


     EQUALIZER DESIGN FOR M=    5
*****************************************


ZERO FORCING EQUALIZER DESIGN
*****************************

THE VALUES OF THE COEFFICIENTS
------------------------------


 C (   1)=  -0.04165
 C (   2)=  -0.08807
 C (   3)=   2.24458
 C (   4)=  -0.08797
 C (   5)=  -0.04162


THE PEAK DISTORTION                 0.1004505
THE MEAN SQUARE DISTORTION          0.0009251
THE MEAN SQUARE ERROR               0.0009251
```

THE EQUALIZED STREAM OF DATA

```
Y(  1) =    0.00015
Y(  2) =    0.00033
Y(  3) =   -0.00801
Y(  4) =    0.00022
Y(  5) =   -0.00868
Y(  6) =   -0.00120
Y(  7) =   -0.01106
Y(  8) =    0.01106
Y(  9) =   -0.01106
Y( 10) =   -0.98893
Y( 11) =    0.98893
Y( 12) =   -0.98893
Y( 13) =    0.97669
Y( 14) =   -0.98656
Y( 15) =    0.97810
Y( 16) =   -0.98574
Y( 17) =    0.97824
Y( 18) =   -0.98628
Y( 19) =    0.99443
Y( 20) =   -0.98649
Y( 21) =    0.99494
Y( 22) =   -0.98507
Y( 23) =    0.99733
Y( 24) =   -0.99733
Y( 25) =    0.99733
Y( 26) =    0.00267
Y( 27) =   -0.00267
Y( 28) =    0.00267
Y( 29) =    0.00958
Y( 30) =    0.00029
Y( 31) =    0.00817
Y( 32) =   -0.00053
Y( 33) =    0.00788
Y( 34) =   -0.00032
Y( 35) =   -0.00015
```

EQUALIZER WITH MINIMUM MEAN SQUARE ERROR
****

THE VALUES OF THE COFFICIENTS

```
C (  1)=  -0.04423
C (  2)=  -0.08989
C (  3)=   2.24278
C (  4)=  -0.08979
C (  5)=  -0.04419
```

```
THE PEAK DISTORTION                 0.1043367
THE MEAN SQUARE DISTORTION          0.0009232
THE MEAN SQUARE ERROR               0.0009240
```

```
THE EQUALIZED  INPUT STREAM
---------------------------
```

```
Y(  1) =    0.00016
Y(  2) =    0.00034
Y(  3) =   -0.00799
Y(  4) =    0.00023
Y(  5) =   -0.00864
Y(  6) =   -0.00117
Y(  7) =   -0.01100
Y(  8) =    0.01223
Y(  9) =   -0.01130
Y( 10) =   -0.98778
Y( 11) =    0.98871
Y( 12) =   -0.98748
Y( 13) =    0.97533
Y( 14) =   -0.98514
Y( 15) =    0.97673
Y( 16) =   -0.98432
Y( 17) =    0.97685
Y( 18) =   -0.98488
Y( 19) =    0.99302
Y( 20) =   -0.98510
Y( 21) =    0.99351
Y( 22) =   -0.98370
Y( 23) =    0.99587
Y( 24) =   -0.99710
Y( 25) =    0.99617
Y( 26) =    0.00291
Y( 27) =   -0.00384
Y( 28) =    0.00261
Y( 29) =    0.00954
Y( 30) =    0.00027
Y( 31) =    0.00814
Y( 32) =   -0.00055
Y( 33) =    0.00786
Y( 34) =   -0.00032
```

Y(35) = -0.00016

APPENDIX V

TABLE(4.1):Pe vs. S/N due to AWGN for Coherent Binary Digital Modulation Schemes

S/N	ASK	FSK	PSK
0.00000	0.231756E 00	0.134680E 00	0.786490E-01
4.77121	0.110335E 00	0.278668E-01	0.715294E-02
6.98970	0.569231E-01	0.675909E-02	0.782700E-03
8.45098	0.306844E-01	0.173713E-02	0.914050E-04
9.54242	0.169474E-01	0.460501E-03	0.110452E-04
10.41392	0.950824E-02	0.124478E-03	0.136378E-05
11.13943	0.539374E-02	0.341026E-04	0.170817E-06
11.76091	0.308494E-02	0.943532E-05	0.312694E-07
12.30449	0.177575E-02	0.263029E-05	0.409467E-08
12.78753	0.102737E-02	0.737951E-06	0.537357E-09
13.22219	0.596875E-03	0.207981E-06	0.706339E-10
13.61728	0.347983E-03	0.610004E-07	0.929617E-11
13.97940	0.203475E-03	0.242489E-07	0.122464E-11
14.31363	0.119282E-03	0.701601E-08	0.161459E-12
14.62398	0.700799E-04	0.203168E-08	0.212998E-13
14.91362	0.412527E-04	0.588762E-09	0.281130E-14
15.18514	0.243250E-04	0.170721E-09	0.371206E-15
15.44067	0.143655E-04	0.495288E-10	0.490305E-16
15.68201	0.849520E-05	0.143754E-10	0.647788E-17
15.91064	0.503002E-05	0.417395E-11	0.856039E-18
16.12782	0.298158E-05	0.121232E-11	0.113143E-18
16.33467	0.176978E-05	0.352217E-12	0.149562E-19
16.53210	0.105116E-05	0.102356E-12	0.197724E-20
16.72096	0.624865E-06	0.297515E-13	0.261414E-21
16.90195	0.371745E-06	0.864947E-14	0.345631E-22
17.07570	0.221324E-06	0.251502E-14	0.457005E-23
17.24275	0.136300E-06	0.731420E-15	0.604264E-24
17.40361	0.813966E-07	0.212736E-15	0.798964E-25
17.55875	0.486359E-07	0.618822E-16	0.105637E-25
17.70851	0.403244E-07	0.180026E-16	0.139665E-26

* S/N in dB

TABLE(4.2):Pe vs. S/N due to AWGN for Non-Coherent
Binary Digital Modulation Schemes

S/N	N-ASK	N-FSK	DPSK
0.00000	0.622166E 00	0.389400E 00	0.183940E 00
4.77121	0.415855E 00	0.236183E 00	0.248935E-01
6.98970	0.286906E 00	0.143252E 00	0.336897E-02
8.45098	0.211763E 00	0.868869E-01	0.455941E-03
9.54242	0.162692E 00	0.526996E-01	0.617049E-04
10.41392	0.126445E 00	0.319639E-01	0.835085E-05
11.13943	0.984569E-01	0.193871E-01	0.113016E-05
11.76091	0.766774E-01	0.117589E-01	0.152951E-06
12.30449	0.597165E-01	0.713212E-02	0.206997E-07
12.78753	0.465072E-01	0.432584E-02	0.280140E-08
13.22219	0.362199E-01	0.262376E-02	0.379128E-09
13.61728	0.282081E-01	0.159139E-02	0.513094E-10
13.97940	0.219685E-01	0.965227E-03	0.694397E-11
14.31363	0.171091E-01	0.585440E-03	0.939764E-12
14.62398	0.133245E-01	0.355087E-03	0.127183E-12
14.91362	0.103772E-01	0.215371E-03	0.172124E-13
15.18514	0.808175E-02	0.130629E-03	0.232944E-14
15.44067	0.629407E-02	0.792307E-04	0.315256E-15
15.68201	0.490183E-02	0.480558E-04	0.426652E-16
15.91064	0.381755E-02	0.291473E-04	0.577411E-17
16.12782	0.297311E-02	0.176787E-04	0.781441E-18
16.33467	0.231546E-02	0.107227E-04	0.105757E-18
16.53210	0.180328E-02	0.650365E-05	0.143126E-19
16.72096	0.140440E-02	0.394466E-05	0.193700E-20
16.90195	0.109375E-02	0.239256E-05	0.262144E-21
17.07570	0.851810E-03	0.145116E-05	0.354774E-22

* S/N in dB

TABLE(4.3):Pe vs. S/N for Binary ASK and PSK with Flat-Fading parameter beta = 0.0

S/N	ASK	PSK
0.00000	0.4393910E 00	0.1037768E 00
4.77121	0.1538663E 00	0.8108694E-02
6.98970	0.7228887E-01	0.8500358E-03
8.45098	0.3705616E-01	0.9722642E-04
9.54242	0.1982171E-01	0.1160442E-04
10.41392	0.1087473E-01	0.1420558E-05
11.13943	0.6067310E-02	0.1768458E-06
11.76091	0.3425902E-02	0.2228087E-07
12.30449	0.1951860E-02	0.2832462E-08
12.78753	0.1119823E-02	0.3625957E-09
13.22219	0.6460543E-03	0.4667680E-10
13.61728	0.3744275E-03	0.6036120E-11
13.97940	0.2178283E-03	0.7835430E-12
14.31363	0.1271321E-03	0.1020380E-12
14.62398	0.7440307E-04	0.1332465E-13
14.91362	0.4364774E-04	0.1744155E-14
15.18514	0.2565890E-04	0.2287808E-15
15.44067	0.1511173E-04	0.3006449E-16
15.68201	0.8914560E-05	0.3957292E-17
15.91064	0.5266492E-05	0.5216482E-18

* S/N in dB

TABLE(4.4):Pe vs. S/N for Binary ASK and PSK with Flat-Fading parameter beta = 0.1

S/N	ASK	PSK
0.00000	0.4407637E 00	0.1058593E 00
4.77121	0.1559347E 00	0.9109370E-02
6.98970	0.7433295E-01	0.1124236E-02
8.45098	0.3885798E-01	0.1596450E-03
9.54242	0.2129517E-01	0.2461599E-04
10.41392	0.1202323E-01	0.4006732E-05
11.13943	0.6933030E-02	0.6771047E-06
11.76091	0.4062563E-02	0.1175559E-06
12.30449	0.2411321E-02	0.2082362E-07
12.78753	0.1446510E-02	0.3745867E-08
13.22219	0.8755813E-03	0.6205398E-09
13.61728	0.5341256E-03	0.1117599E-09
13.97940	0.3280465E-03	0.2029432E-10
14.31363	0.2026873E-03	0.3712355E-11
14.62398	0.1259012E-03	0.6836183E-12
14.91362	0.7857788E-04	0.1266587E-12
15.18514	0.4925256E-04	0.2360044E-13
15.44067	0.3101636E-04	0.5537749E-14
15.68201	0.1956863E-04	0.1056814E-14
15.91064	0.1239554E-04	0.2022903E-15

* S/N in dB

TABLE(4.5):Pe vs. S/N for Binary ASK and PSK with Flat-Fading parameter beta = 0.3

S/N	ASK	PSK
0.00000	0.4517769E 00	0.1230948E 00
4.77121	0.1724048E 00	0.1933675E-01
6.98970	0.9153473E-01	0.5156968E-02
8.45098	0.5467361E-01	0.1488338E-02
9.54242	0.3493547E-01	0.4884987E-03
10.41392	0.2335248E-01	0.1702446E-03
11.13943	0.1612239E-01	0.8028965E-04
11.76091	0.1140225E-01	0.3125671E-04
12.30449	0.8214552E-02	0.1238926E-04
12.78753	0.6004602E-02	0.4980066E-05
13.22219	0.4440583E-02	0.2024316E-05
13.61728	0.3315240E-02	0.8304098E-06
13.97940	0.2494723E-02	0.3432473E-06
14.31363	0.1719682E-02	0.1427966E-06
14.62398	0.1289778E-02	0.5358232E-07
14.91362	0.9721038E-03	0.2213622E-07
15.18514	0.7359504E-03	0.9185772E-08
15.44067	0.5594643E-03	0.3827598E-08
15.68201	0.4269292E-03	0.1601079E-08
15.91064	0.3269599E-03	0.6721834E-09

* S/N in dB

TABLE(4.6):Pe vs. S/N due to AWGN for M-ary
PSK Digital Modulations Approximately

S/N	2-PSK	4-PSK	8-PSK	16-PSK
0.00000	0.36788E 00	0.60654E 00	0.86378E 00	0.96266E 00
3.01030	0.13534E 00	0.36789E 00	0.74611E 00	0.92671E 00
4.77121	0.49787E-01	0.22314E 00	0.64447E 00	0.89210E 00
6.02060	0.18316E-01	0.13535E 00	0.55668E 00	0.85879E 00
6.98970	0.67380E-02	0.82093E-01	0.48085E 00	0.82672E 00
7.78151	0.24788E-02	0.49793E-01	0.41535E 00	0.79585E 00
8.45098	0.91188E-03	0.30201E-01	0.35877E 00	0.76613E 00
9.03090	0.33546E-03	0.18318E-01	0.30990E 00	0.73752E 00
9.54242	0.12341E-03	0.11111E-01	0.26768E 00	0.70997E 00
10.00000	0.45400E-04	0.67392E-02	0.23122E 00	0.68346E 00
10.41392	0.16702E-04	0.40876E-02	0.19972E 00	0.65794E 00
10.79181	0.61442E-05	0.24793E-02	0.17251E 00	0.63337E 00
11.13943	0.22603E-05	0.15038E-02	0.14901E 00	0.60972E 00
11.46128	0.83153E-06	0.91212E-03	0.12871E 00	0.58695E 00
11.76091	0.30590E-06	0.55324E-03	0.11118E 00	0.56503E 00
12.04120	0.11254E-06	0.33556E-03	0.96035E-01	0.54393E 00
12.30449	0.41400E-07	0.20353E-03	0.82953E-01	0.52362E 00
12.55272	0.15230E-07	0.12345E-03	0.71653E-01	0.50406E 00
12.78753	0.56029E-08	0.74878E-04	0.61892E-01	0.48524E 00
13.01029	0.20612E-08	0.45417E-04	0.53461E-01	0.46712E 00
13.22219	0.75827E-09	0.27547E-04	0.46179E-01	0.44968E 00
13.42422	0.27895E-09	0.16709E-04	0.39888E-01	0.43288E 00
13.61728	0.10262E-09	0.10134E-04	0.34454E-01	0.41672E 00
13.80211	0.37752E-10	0.61470E-05	0.29761E-01	0.40116E 00
13.97940	0.13888E-10	0.37284E-05	0.25707E-01	0.38618E 00
14.14973	0.51092E-11	0.22614E-05	0.22205E-01	0.37176E 00
14.31363	0.18796E-11	0.13716E-05	0.19180E-01	0.35787E 00
14.47158	0.69145E-12	0.83196E-06	0.16567E-01	0.34451E 00
14.62398	0.25437E-12	0.50462E-06	0.14311E-01	0.33164E 00
14.77120	0.93578E-13	0.30607E-06	0.12361E-01	0.31926E 00

* S/N in dB

TABLE(4.7):Pe vs. S/N due to Flat-Fading for M-ary PSK Modulations Approximately with beta = 0.0

S/N	2-PSK	4-PSK	8-PSK
11.76091	0.19220E-05	0.34751E-02	0.69850E 00
12.04120	0.70708E-06	0.21078E-02	0.60334E 00
12.30449	0.26012E-06	0.12784E-02	0.52115E 00
12.55272	0.95694E-07	0.77540E-03	0.45016E 00
12.78753	0.35204E-07	0.47030E-03	0.38883E 00
13.01029	0.12951E-07	0.28525E-03	0.33586E 00
13.22219	0.47643E-08	0.17302E-03	0.29011E 00
13.42422	0.17527E-08	0.10494E-03	0.25059E 00
13.61728	0.64478E-09	0.63649E-04	0.21645E 00
13.80211	0.23720E-09	0.38605E-04	0.18696E 00
13.97940	0.87262E-10	0.23415E-04	0.16149E 00
14.14973	0.32102E-10	0.14202E-04	0.13949E 00
14.31363	0.11810E-10	0.86139E-05	0.12049E 00
14.47158	0.43445E-11	0.52246E-05	0.10408E 00
14.62398	0.15983E-11	0.31689E-05	0.89898E-01
14.77120	0.58797E-12	0.19220E-05	0.77651E-01
14.91362	0.21630E-12	0.11658E-05	0.67073E-01
15.05150	0.79572E-13	0.70707E-06	0.57936E-01
15.18514	0.29273E-13	0.42886E-06	0.50043E-01
15.31479	0.10769E-13	0.26012E-06	0.43226E-01
15.44067	0.39617E-14	0.15777E-06	0.37338E-01

* S/N in dB

TABLE(4.8):Pe vs. S/N due to Flat-Fading for M-ary PSK Modulations Approximately with beta = 0.1

S/N	2-PSK	4-PSK	8-PSK
11.76091	0.84154E-05	0.54715E-02	0.72444E 00
12.04120	0.36119E-05	0.35134E-02	0.62933E 00
12.30449	0.15602E-05	0.22610E-02	0.54692E 00
12.55272	0.67311E-06	0.14580E-02	0.47548E 00
12.78753	0.29096E-06	0.94196E-03	0.41353E 00
13.01029	0.12598E-06	0.60959E-03	0.35979E 00
13.22219	0.54628E-07	0.39545E-03	0.31315E 00
13.42422	0.23719E-07	0.25648E-03	0.27265E 00
13.61728	0.10311E-07	0.16671E-03	0.23748E 00
13.80211	0.44870E-08	0.10849E-03	0.20692E 00
13.97940	0.19545E-08	0.70683E-04	0.18035E 00
14.14973	0.85206E-09	0.46098E-04	0.15725E 00
14.31363	0.37175E-09	0.30094E-04	0.13716E 00
14.47158	0.16231E-09	0.19663E-04	0.11967E 00
14.62398	0.70912E-10	0.12859E-04	0.10445E 00
14.77120	0.30999E-10	0.84154E-05	0.91189E-01
14.91362	0.13559E-10	0.55113E-05	0.79640E-01
15.05150	0.59333E-11	0.36118E-05	0.69575E-01
15.18514	0.25977E-11	0.23771E-05	0.60801E-01
15.31479	0.11378E-11	0.15601E-05	0.53150E-01
15.44067	0.49853E-12	0.10245E-05	0.46475E-01

* S/N in dB

TABLE(4.9):Pe vs. S/N due to Flat-Fading for M-ary PSK Modulations Approximately with beta = 0.3

S/N	2-PSK	4-PSK	8-PSK
11.76091	0.51449E-03	0.35751E-01	0.92550E 00
12.04120	0.37952E-03	0.27081E-01	0.83200E 00
12.30449	0.22557E-03	0.20556E-01	0.74932E 00
12.55272	0.13431E-03	0.15632E-01	0.67600E 00
12.78753	0.80091E-04	0.11907E-01	0.61083E 00
13.01029	0.47826E-04	0.81897E-02	0.55274E 00
13.22219	0.28632E-04	0.61771E-02	0.50087E 00
13.42422	0.17144E-04	0.46649E-02	0.45444E 00
13.61728	0.10276E-04	0.35272E-02	0.41279E 00
13.80211	0.61650E-05	0.26701E-02	0.37538E 00
13.97940	0.37021E-05	0.20236E-02	0.34170E 00
14.14973	0.22249E-05	0.15354E-02	0.31133E 00
14.31363	0.13381E-05	0.11663E-02	0.28392E 00
14.47158	0.80535E-06	0.88686E-03	0.25912E 00
14.62398	0.48501E-06	0.67512E-03	0.23667E 00
14.77120	0.27958E-06	0.51449E-03	0.21632E 00
14.91362	0.16756E-06	0.39249E-03	0.19785E 00
15.05150	0.10046E-06	0.37952E-03	0.18107E 00
15.18514	0.60251E-07	0.29252E-03	0.16581E 00
15.31479	0.36149E-07	0.22557E-03	0.15208E 00
15.44067	0.21696E-07	0.17402E-03	0.13926E 00

* S/N in dB

TABLE(4.10):Pe vs. S/N due to Flat-Fading for 4-QAM Modulation

S/N	beta=0.0	beta=0.1	beta=0.3
10.00000	0.16993E-02	0.21975E-02	0.79296E-02
10.79181	0.57085E-03	0.81250E-03	0.40843E-02
11.46128	0.19444E-03	0.30801E-03	0.21599E-02
12.04120	0.66914E-04	0.11909E-03	0.11645E-02
12.55272	0.23209E-04	0.46815E-04	0.57598E-03
13.01029	0.80999E-05	0.18639E-04	0.31106E-03
13.42422	0.28411E-05	0.75020E-05	0.16966E-03
13.80211	0.10007E-05	0.30471E-05	0.93362E-04
14.14973	0.35369E-06	0.12472E-05	0.51788E-04
14.47158	0.12538E-06	0.51388E-06	0.28938E-04
14.77120	0.44562E-07	0.21292E-06	0.20558E-04
15.05150	0.15873E-07	0.88657E-07	0.11833E-04
15.31479	0.56650E-08	0.37073E-07	0.68349E-05
15.56302	0.20253E-08	0.15561E-07	0.39612E-05
15.79783	0.72520E-09	0.65533E-08	0.23026E-05
16.02058	0.26003E-09	0.27680E-08	0.13420E-05
16.23248	0.93355E-10	0.11722E-08	0.78409E-06
16.43452	0.33554E-10	0.49762E-09	0.45527E-06
16.62756	0.12072E-10	0.21169E-09	0.26597E-06
16.81241	0.43477E-11	0.90232E-10	0.15562E-06
16.98969	0.15671E-11	0.38529E-10	0.91177E-07
17.16002	0.56531E-12	0.16478E-10	0.53490E-07
17.32393	0.20408E-12	0.70579E-11	0.31418E-07
17.48187	0.73724E-13	0.30272E-11	0.18475E-07
17.63428	0.26650E-13	0.13000E-11	0.10875E-07
17.78149	0.96391E-14	0.55892E-12	0.64079E-08
17.92390	0.34884E-14	0.24056E-12	0.37793E-08
18.06178	0.12631E-14	0.10364E-12	0.22309E-08
18.19543	0.45757E-15	0.44693E-13	0.13181E-08
18.32507	0.16584E-15	0.19290E-13	0.77934E-09
18.45097	0.60130E-16	0.83322E-14	0.46116E-09

* S/N in dB

TABLE(4.11):Pe vs. S/N due to Flat-Fading for 16-QAM Modulation

S/N	beta=0.0	beta=0.1	beta=0.3
10.00000	0.28710E 00	0.29237E 00	0.33573E 00
10.79181	0.21915E 00	0.22439E 00	0.26751E 00
11.46128	0.16860E 00	0.17370E 00	0.21580E 00
12.04120	0.13052E 00	0.13540E 00	0.17591E 00
12.55272	0.10155E 00	0.10616E 00	0.14466E 00
13.01029	0.79347E-01	0.83646E-01	0.11989E 00
13.42422	0.62221E-01	0.66192E-01	0.10002E 00
13.80211	0.48943E-01	0.52580E-01	0.83943E-01
14.14973	0.38602E-01	0.41908E-01	0.70816E-01
14.47158	0.30518E-01	0.33504E-01	0.60018E-01
14.77120	0.24178E-01	0.26860E-01	0.51078E-01
15.05150	0.19191E-01	0.21588E-01	0.43630E-01
15.31479	0.15258E-01	0.17391E-01	0.37392E-01
15.56302	0.12150E-01	0.14040E-01	0.32142E-01
15.79783	0.96884E-02	0.11357E-01	0.27732E-01
16.02058	0.77351E-02	0.92038E-02	0.23939E-01
16.23248	0.61828E-02	0.74718E-02	0.20731E-01
16.43452	0.49472E-02	0.60756E-02	0.17990E-01
16.62756	0.39624E-02	0.49479E-02	0.15641E-01
16.81241	0.31764E-02	0.40352E-02	0.13622E-01
16.98969	0.25485E-02	0.32954E-02	0.11883E-01
17.16002	0.20463E-02	0.26946E-02	0.10381E-01
17.32393	0.16442E-02	0.22061E-02	0.90808E-02
17.48187	0.13220E-02	0.18082E-02	0.79538E-02
17.63428	0.10636E-02	0.14836E-02	0.69750E-02
17.78149	0.85621E-03	0.12186E-02	0.61233E-02
17.92390	0.68964E-03	0.10019E-02	0.53812E-02
18.06178	0.55575E-03	0.82457E-03	0.47336E-02
18.19543	0.44808E-03	0.67920E-03	0.41677E-02
18.32507	0.36143E-03	0.55994E-03	0.36726E-02
18.45097	0.29166E-03	0.46200E-03	0.32389E-02

* S/N in dB

TABLE(4.12):Pe vs. S/N due to Flat-Fading for
64-QAM Modulation

S/N	beta=0.0	beta=0.1	beta=0.3
17.78149	0.18817E 00	0.19399E 00	0.24199E 00
18.06178	0.16667E 00	0.17240E 00	0.21965E 00
18.32507	0.14780E 00	0.15340E 00	0.19977E 00
18.57332	0.13119E 00	0.13666E 00	0.18204E 00
18.80812	0.11656E 00	0.12187E 00	0.16617E 00
19.03088	0.10366E 00	0.10881E 00	0.15193E 00
19.24278	0.92252E-01	0.97232E-01	0.13912E 00
19.44481	0.82165E-01	0.86969E-01	0.12759E 00
19.63786	0.73232E-01	0.77857E-01	0.11717E 00
19.82271	0.65314E-01	0.69757E-01	0.10774E 00
19.99998	0.58289E-01	0.62549E-01	0.99188E-01
20.17032	0.52049E-01	0.56127E-01	0.91424E-01
20.33423	0.46503E-01	0.50400E-01	0.84359E-01
20.49217	0.41570E-01	0.45288E-01	0.77922E-01
20.64458	0.37178E-01	0.40721E-01	0.72046E-01
20.79179	0.33266E-01	0.36637E-01	0.66676E-01
20.93420	0.29779E-01	0.32983E-01	0.61761E-01
21.07208	0.26668E-01	0.29709E-01	0.57257E-01
21.20572	0.23892E-01	0.26776E-01	0.53123E-01
21.33537	0.21414E-01	0.24144E-01	0.49326E-01
21.46127	0.19199E-01	0.21783E-01	0.45833E-01
21.58362	0.17219E-01	0.19661E-01	0.42618E-01
21.70261	0.15449E-01	0.17755E-01	0.39653E-01
21.81842	0.13865E-01	0.16041E-01	0.36918E-01
21.93123	0.12447E-01	0.14499E-01	0.34425E-01
22.04118	0.11178E-01	0.13111E-01	0.32091E-01
22.14844	0.10040E-01	0.11860E-01	0.29899E-01
22.25308	0.90213E-02	0.10734E-01	0.27900E-01
22.35527	0.81078E-02	0.97175E-02	0.26047E-01
22.45511	0.72885E-02	0.88010E-02	0.24330E-01
22.55272	0.65536E-02	0.79738E-02	0.22736E-01
22.64816	0.58941E-02	0.72269E-02	0.21256E-01
22.74156	0.53021E-02	0.65522E-02	0.19880E-01
22.83299	0.47706E-02	0.59425E-02	0.18602E-01
22.92255	0.42933E-02	0.53914E-02	0.17412E-01
23.01028	0.38644E-02	0.48928E-02	0.16305E-01

* S/N in db

TABLE(4.13):Pe vs. S/N due to AWGN for
M-QAM Modulation

S/N	4-QAM	16-QAM	64-QAM	256-QAM
0.00000	0.29214E 00	0.74096E 00	0.92374E 00	0.97941E 00
4.77121	0.81532E-01	0.53422E 00	0.85352E 00	0.95911E 00
6.98970	0.25187E-01	0.41933E 00	0.79514E 00	0.94136E 00
8.45098	0.81344E-02	0.32356E 00	0.74319E 00	0.92479E 00
9.54242	0.26980E-02	0.25140E 00	0.69596E 00	0.90899E 00
10.41392	0.91091E-03	0.19630E 00	0.63606E 00	0.89379E 00
11.13943	0.31147E-03	0.15387E 00	0.59589E 00	0.87907E 00
11.76091	0.10751E-03	0.12100E 00	0.57525E 00	0.86478E 00
12.30449	0.37380E-04	0.95232E-01	0.54062E 00	0.85085E 00
12.78753	0.13072E-04	0.75401E-01	0.50834E 00	0.83726E 00
13.22219	0.45944E-05	0.59717E-01	0.47820E 00	0.82398E 00
13.61728	0.16207E-05	0.47383E-01	0.45003E 00	0.81100E 00
13.97940	0.57363E-06	0.37660E-01	0.42366E 00	0.79828E 00
14.31363	0.21064E-06	0.29977E-01	0.39897E 00	0.78582E 00
14.62398	0.10401E-06	0.23895E-01	0.37581E 00	0.77362E 00
14.91362	0.37611E-07	0.19071E-01	0.35410E 00	0.76165E 00
15.18514	0.13611E-07	0.15238E-01	0.33373E 00	0.74990E 00
15.44067	0.49281E-08	0.12189E-01	0.31461E 00	0.73838E 00
15.68201	0.17853E-08	0.97597E-02	0.29665E 00	0.71079E 00
15.91064	0.64703E-09	0.78216E-02	0.27977E 00	0.69948E 00
16.12782	0.23458E-09	0.62737E-02	0.26390E 00	0.68841E 00
16.33467	0.85079E-10	0.50361E-02	0.24898E 00	0.67757E 00
16.53210	0.30865E-10	0.40456E-02	0.23495E 00	0.66694E 00
16.72096	0.11200E-10	0.32521E-02	0.22175E 00	0.65651E 00
16.90195	0.40652E-11	0.26160E-02	0.20933E 00	0.64629E 00
17.07570	0.14758E-11	0.21055E-02	0.19763E 00	0.63627E 00
17.24275	0.53586E-12	0.16956E-02	0.18662E 00	0.62644E 00
17.40361	0.19460E-12	0.13662E-02	0.17624E 00	0.61679E 00
17.55875	0.70681E-13	0.11014E-02	0.16647E 00	0.60732E 00
17.70851	0.25676E-13	0.88827E-03	0.15726E 00	0.59803E 00
17.85329	0.93280E-14	0.71672E-03	0.14859E 00	0.60575E 00
17.99339	0.33892E-14	0.57854E-03	0.14041E 00	0.59671E 00
18.12912	0.12316E-14	0.46718E-03	0.13269E 00	0.58783E 00
18.26074	0.44755E-15	0.37740E-03	0.12542E 00	0.57908E 00
18.38849	0.16266E-15	0.30498E-03	0.11835E 00	0.57048E 00

* S/N in dB

TABLE(4.14):Pe vs. S/N due to Frequency-
Selective Fading for 4-QAM Modulation
with delta = -0.7 and tau/T variable

S/N	t/T=0.0	t/T=0.2	t/T=0.5	t/T=0.7	t/T=1.0
0.0	0.618E 00	0.553E 00	0.468E 00	0.424E 00	0.381E 00
4.8	0.512E 00	0.406E 00	0.314E 00	0.290E 00	0.280E 00
7.0	0.427E 00	0.311E 00	0.239E 00	0.230E 00	0.228E 00
8.5	0.370E 00	0.251E 00	0.199E 00	0.196E 00	0.196E 00
9.5	0.334E 00	0.212E 00	0.177E 00	0.176E 00	0.176E 00
10.4	0.294E 00	0.178E 00	0.154E 00	0.153E 00	0.153E 00
11.1	0.260E 00	0.152E 00	0.135E 00	0.135E 00	0.135E 00
11.8	0.230E 00	0.130E 00	0.119E 00	0.119E 00	0.119E 00
12.3	0.204E 00	0.113E 00	0.105E 00	0.105E 00	0.105E 00
12.8	0.182E 00	0.987E-01	0.932E-01	0.932E-01	0.932E-01
13.2	0.162E 00	0.866E-01	0.828E-01	0.828E-01	0.828E-01
13.6	0.145E 00	0.763E-01	0.737E-01	0.737E-01	0.737E-01
14.0	0.129E 00	0.675E-01	0.657E-01	0.657E-01	0.657E-01
14.3	0.115E 00	0.599E-01	0.586E-01	0.586E-01	0.586E-01
14.6	0.103E 00	0.533E-01	0.524E-01	0.524E-01	0.524E-01
14.9	0.926E-01	0.475E-01	0.469E-01	0.469E-01	0.469E-01
15.2	0.830E-01	0.424E-01	0.420E-01	0.420E-01	0.420E-01
15.4	0.745E-01	0.379E-01	0.376E-01	0.376E-01	0.376E-01
15.7	0.668E-01	0.339E-01	0.337E-01	0.337E-01	0.337E-01
15.9	0.601E-01	0.304E-01	0.303E-01	0.303E-01	0.303E-01
16.1	0.540E-01	0.273E-01	0.272E-01	0.272E-01	0.272E-01
16.3	0.486E-01	0.245E-01	0.244E-01	0.244E-01	0.244E-01
16.5	0.437E-01	0.220E-01	0.220E-01	0.220E-01	0.220E-01
16.7	0.393E-01	0.198E-01	0.198E-01	0.198E-01	0.198E-01
16.9	0.354E-01	0.178E-01	0.178E-01	0.178E-01	0.178E-01
17.1	0.319E-01	0.160E-01	0.160E-01	0.160E-01	0.160E-01
17.2	0.288E-01	0.144E-01	0.144E-01	0.144E-01	0.144E-01
17.4	0.259E-01	0.130E-01	0.130E-01	0.130E-01	0.130E-01
17.6	0.234E-01	0.117E-01	0.117E-01	0.117E-01	0.117E-01
17.7	0.211E-01	0.106E-01	0.106E-01	0.106E-01	0.106E-01

* S/N in dB
* t/T stands for tau/T

TABLE(4.15):Pe vs. S/N due to Frequency-
Selective Fading for 4-QAM Modulation
with delta = -0.2 and tau/T variable

S/N	t/T=0.0	t/T=0.2	t/T=0.5	t/T=0.7	t/T=1.0
0.0	0.367E 00	0.355E 00	0.330E 00	0.315E 00	0.294E 00
4.8	0.159E 00	0.141E 00	0.121E 00	0.110E 00	0.992E-01
7.0	0.723E-01	0.604E-01	0.489E-01	0.442E-01	0.401E-01
8.5	0.340E-01	0.269E-01	0.211E-01	0.192E-01	0.178E-01
9.5	0.163E-01	0.123E-01	0.952E-02	0.878E-02	0.834E-02
10.4	0.796E-02	0.574E-02	0.444E-02	0.415E-02	0.402E-02
11.1	0.392E-02	0.271E-02	0.212E-02	0.201E-02	0.197E-02
11.8	0.194E-02	0.130E-02	0.103E-02	0.987E-03	0.974E-03
12.3	0.972E-03	0.629E-03	0.505E-03	0.490E-03	0.486E-03
12.8	0.488E-03	0.307E-03	0.251E-03	0.245E-03	0.244E-03
13.2	0.246E-03	0.151E-03	0.125E-03	0.124E-03	0.123E-03
13.6	0.125E-03	0.746E-04	0.632E-04	0.625E-04	0.624E-04
14.0	0.633E-04	0.371E-04	0.320E-04	0.317E-04	0.317E-04
14.3	0.323E-04	0.185E-04	0.162E-04	0.161E-04	0.161E-04
14.6	0.165E-04	0.931E-05	0.827E-05	0.824E-05	0.823E-05
14.9	0.842E-05	0.469E-05	0.422E-05	0.421E-05	0.421E-05
15.2	0.431E-05	0.237E-05	0.216E-05	0.216E-05	0.216E-05
15.4	0.221E-05	0.120E-05	0.111E-05	0.111E-05	0.111E-05
15.7	0.114E-05	0.612E-06	0.569E-06	0.569E-06	0.569E-06
15.9	0.585E-06	0.312E-06	0.293E-06	0.293E-06	0.293E-06
16.1	0.302E-06	0.160E-06	0.151E-06	0.151E-06	0.151E-06
16.3	0.155E-06	0.817E-07	0.778E-07	0.777E-07	0.777E-07
16.5	0.803E-07	0.419E-07	0.401E-07	0.401E-07	0.401E-07
16.7	0.415E-07	0.216E-07	0.207E-07	0.207E-07	0.207E-07
16.9	0.215E-07	0.113E-07	0.107E-07	0.107E-07	0.107E-07
17.1	0.111E-07	0.580E-08	0.555E-08	0.555E-08	0.555E-08
17.2	0.575E-08	0.299E-08	0.287E-08	0.287E-08	0.287E-08
17.4	0.298E-08	0.154E-08	0.149E-08	0.149E-08	0.149E-08
17.6	0.162E-08	0.835E-09	0.811E-09	0.811E-09	0.811E-09
17.7	0.843E-09	0.432E-09	0.422E-09	0.422E-09	0.422E-09

* S/N in dB
* t/T stands for tau/T

TABLE(4.16):Pe vs. S/N due to Frequency-
Selective Fading for 4-QAM Modulation
with delta = 0.0 and tau/T variable

S/N	t/T=0.0	t/T=0.2	t/T=0.5	t/T=0.7	t/T=1.0
0.0	0.292E 00	0.292E 00	0.292E 00	0.292E 00	0.292E 00
4.8	0.815E-01	0.815E-01	0.815E-01	0.815E-01	0.815E-01
7.0	0.252E-01	0.252E-01	0.252E-01	0.252E-01	0.252E-01
8.5	0.813E-02	0.813E-02	0.813E-02	0.813E-02	0.813E-02
9.5	0.270E-02	0.270E-02	0.270E-02	0.270E-02	0.270E-02
10.4	0.911E-03	0.911E-03	0.911E-03	0.911E-03	0.911E-03
11.1	0.311E-03	0.311E-03	0.311E-03	0.311E-03	0.311E-03
11.8	0.108E-03	0.108E-03	0.108E-03	0.108E-03	0.108E-03
12.3	0.374E-04	0.374E-04	0.374E-04	0.374E-04	0.374E-04
12.8	0.131E-04	0.131E-04	0.131E-04	0.131E-04	0.131E-04
13.2	0.459E-05	0.459E-05	0.459E-05	0.459E-05	0.459E-05
13.6	0.162E-05	0.162E-05	0.162E-05	0.162E-05	0.162E-05
14.0	0.573E-06	0.573E-06	0.573E-06	0.573E-06	0.573E-06
14.3	0.203E-06	0.203E-06	0.203E-06	0.203E-06	0.203E-06
14.6	0.724E-07	0.724E-07	0.724E-07	0.724E-07	0.724E-07
14.9	0.258E-07	0.258E-07	0.258E-07	0.258E-07	0.258E-07
15.2	0.923E-08	0.923E-08	0.923E-08	0.923E-08	0.923E-08
15.4	0.330E-08	0.330E-08	0.330E-08	0.330E-08	0.330E-08
15.7	0.124E-08	0.124E-08	0.124E-08	0.124E-08	0.124E-08
15.9	0.647E-09	0.647E-09	0.647E-09	0.647E-09	0.647E-09
16.1	0.235E-09	0.235E-09	0.235E-09	0.235E-09	0.235E-09
16.3	0.851E-10	0.851E-10	0.851E-10	0.851E-10	0.851E-10
16.5	0.309E-10	0.309E-10	0.309E-10	0.309E-10	0.309E-10
16.7	0.112E-10	0.112E-10	0.112E-10	0.112E-10	0.112E-10
16.9	0.407E-11	0.407E-11	0.407E-11	0.407E-11	0.407E-11
17.1	0.148E-11	0.148E-11	0.148E-11	0.148E-11	0.148E-11
17.2	0.536E-12	0.536E-12	0.536E-12	0.536E-12	0.536E-12
17.4	0.195E-12	0.195E-12	0.195E-12	0.195E-12	0.195E-12
17.6	0.707E-13	0.707E-13	0.707E-13	0.707E-13	0.707E-13
17.7	0.257E-13	0.257E-13	0.257E-13	0.257E-13	0.257E-13

* S/N in dB
* t/T stands for tau/T

TABLE(4.17):Pe vs. S/N due to Frequency-
Selective Fading for 4-QAM Modulation
with delta = 0.2 and tau/T variable

S/N	t/T=0.0	t/T=0.2	t/T=0.5	t/T=0.7	t/T=1.0
0.0	0.217E 00	0.231E 00	0.255E 00	0.272E 00	0.294E 00
4.8	0.373E-01	0.445E-01	0.596E-01	0.730E-01	0.992E-01
7.0	0.728E-02	0.975E-02	0.163E-01	0.233E-01	0.401E-01
8.5	0.150E-02	0.227E-02	0.482E-02	0.820E-02	0.178E-01
9.5	0.318E-03	0.549E-03	0.151E-02	0.305E-02	0.834E-02
10.4	0.689E-04	0.136E-03	0.490E-03	0.117E-02	0.402E-02
11.1	0.151E-04	0.345E-04	0.163E-03	0.462E-03	0.197E-02
11.8	0.336E-05	0.888E-05	0.554E-04	0.185E-03	0.974E-03
12.3	0.751E-06	0.231E-05	0.191E-04	0.747E-04	0.486E-03
12.8	0.169E-06	0.610E-06	0.662E-05	0.304E-04	0.244E-03
13.2	0.382E-07	0.162E-06	0.232E-05	0.125E-04	0.123E-03
13.6	0.867E-08	0.434E-07	0.814E-06	0.512E-05	0.624E-04
14.0	0.207E-08	0.118E-07	0.288E-06	0.211E-05	0.317E-04
14.3	0.688E-09	0.330E-08	0.102E-06	0.875E-06	0.161E-04
14.6	0.160E-09	0.935E-09	0.363E-07	0.363E-06	0.823E-05
14.9	0.370E-10	0.361E-09	0.129E-07	0.151E-06	0.421E-05
15.2	0.861E-11	0.100E-09	0.462E-08	0.629E-07	0.216E-05
15.4	0.200E-11	0.279E-10	0.165E-08	0.262E-07	0.111E-05
15.7	0.465E-12	0.777E-11	0.622E-09	0.110E-07	0.569E-06
15.9	0.108E-12	0.217E-11	0.324E-09	0.459E-08	0.293E-06
16.1	0.252E-13	0.606E-12	0.117E-09	0.192E-08	0.151E-06
16.3	0.586E-14	0.169E-12	0.425E-10	0.847E-09	0.777E-07
16.5	0.136E-14	0.474E-13	0.154E-10	0.514E-09	0.401E-07
16.7	0.317E-15	0.133E-13	0.560E-11	0.218E-09	0.207E-07
16.9	0.739E-16	0.372E-14	0.203E-11	0.922E-10	0.107E-07
17.1	0.172E-16	0.104E-14	0.738E-12	0.391E-10	0.555E-08
17.2	0.401E-17	0.293E-15	0.268E-12	0.166E-10	0.287E-08
17.4	0.933E-18	0.821E-16	0.973E-13	0.703E-11	0.149E-08
17.6	0.217E-18	0.231E-16	0.353E-13	0.298E-11	0.811E-09
17.7	0.506E-19	0.647E-17	0.128E-13	0.126E-11	0.422E-09

* S/N in dB
* t/T stands for tau/T

TABLE(4.18):Pe vs. S/N due to Frequency-
Selective Fading for 4-QAM Modulation
with delta = 0.7 and tau/T variable

S/N	t/T=0.0	t/T=0.2	t/T=0.5	t/T=0.7	t/T=1.0
0.0	0.871E-01	0.119E 00	0.193E 00	0.254E 00	0.381E 00
4.8	0.323E-02	0.856E-02	0.428E-01	0.105E 00	0.280E 00
7.0	0.144E-03	0.820E-03	0.127E-01	0.531E-01	0.228E 00
8.5	0.687E-05	0.894E-04	0.407E-02	0.282E-01	0.196E 00
9.5	0.340E-06	0.104E-04	0.135E-02	0.153E-01	0.176E 00
10.4	0.172E-07	0.125E-05	0.456E-03	0.845E-02	0.153E 00
11.1	0.929E-09	0.153E-06	0.156E-03	0.471E-02	0.135E 00
11.8	0.712E-10	0.191E-07	0.538E-04	0.265E-02	0.119E 00
12.3	0.381E-11	0.239E-08	0.187E-04	0.150E-02	0.105E 00
12.8	0.204E-12	0.459E-09	0.654E-05	0.849E-03	0.932E-01
13.2	0.109E-13	0.593E-10	0.230E-05	0.484E-03	0.828E-01
13.6	0.585E-15	0.768E-11	0.810E-06	0.277E-03	0.737E-01
14.0	0.314E-16	0.995E-12	0.287E-06	0.159E-03	0.657E-01
14.3	0.169E-17	0.129E-12	0.102E-06	0.916E-04	0.586E-01
14.6	0.906E-19	0.167E-13	0.362E-07	0.528E-04	0.524E-01
14.9	0.487E-20	0.217E-14	0.129E-07	0.305E-04	0.469E-01
15.2	0.261E-21	0.282E-15	0.461E-08	0.177E-04	0.420E-01
15.4	0.141E-22	0.367E-16	0.165E-08	0.102E-04	0.376E-01
15.7	0.755E-24	0.477E-17	0.622E-09	0.595E-05	0.337E-01
15.9	0.406E-25	0.619E-18	0.324E-09	0.346E-05	0.303E-01
16.1	0.218E-26	0.805E-19	0.117E-09	0.201E-05	0.272E-01
16.3	0.117E-27	0.105E-19	0.425E-10	0.117E-05	0.244E-01
16.5	0.629E-29	0.136E-20	0.154E-10	0.683E-06	0.220E-01
16.7	0.338E-30	0.177E-21	0.560E-11	0.399E-06	0.198E-01
16.9	0.181E-31	0.230E-22	0.203E-11	0.233E-06	0.178E-01
17.1	0.974E-33	0.299E-23	0.738E-12	0.136E-06	0.160E-01
17.2	0.523E-34	0.389E-24	0.268E-12	0.796E-07	0.144E-01
17.4	0.281E-35	0.506E-25	0.973E-13	0.466E-07	0.130E-01
17.6	0.151E-36	0.658E-26	0.353E-13	0.273E-07	0.117E-01
17.7	0.808E-38	0.856E-27	0.128E-13	0.160E-07	0.106E-01

* S/N in dB
* t/T stands for tau/T

TABLE(4.19):Pe vs. S/N due to Frequency-
Selective Fading for 4-QAM Modulation
with tau/T = 0.1 and delta variable

S/N	d = -0.7	d = -0.2	d = 0.0	d = 0.2	d = 0.7
0.0	0.585E 00	0.364E 00	0.292E 00	0.224E 00	0.101E 00
4.8	0.450E 00	0.150E 00	0.815E-01	0.407E-01	0.506E-02
7.0	0.364E 00	0.659E-01	0.252E-01	0.837E-02	0.315E-03
8.5	0.301E 00	0.300E-01	0.813E-02	0.182E-02	0.218E-04
9.5	0.258E 00	0.140E-01	0.270E-02	0.410E-03	0.160E-05
10.4	0.219E 00	0.664E-02	0.911E-03	0.942E-04	0.123E-06
11.1	0.186E 00	0.319E-02	0.311E-03	0.220E-04	0.977E-08
11.8	0.160E 00	0.154E-02	0.108E-03	0.520E-05	0.837E-09
12.3	0.138E 00	0.753E-03	0.374E-04	0.124E-05	0.991E-10
12.8	0.119E 00	0.369E-03	0.131E-04	0.298E-06	0.834E-11
13.2	0.104E 00	0.182E-03	0.459E-05	0.722E-07	0.705E-12
13.6	0.904E-01	0.904E-04	0.162E-05	0.176E-07	0.597E-13
14.0	0.791E-01	0.450E-04	0.573E-06	0.436E-08	0.507E-14
14.3	0.694E-01	0.225E-04	0.203E-06	0.122E-08	0.431E-15
14.6	0.610E-01	0.113E-04	0.724E-07	0.400E-09	0.367E-16
14.9	0.538E-01	0.567E-05	0.258E-07	0.100E-09	0.313E-17
15.2	0.476E-01	0.286E-05	0.923E-08	0.252E-10	0.266E-18
15.4	0.421E-01	0.144E-05	0.330E-08	0.633E-11	0.227E-19
15.7	0.373E-01	0.730E-06	0.124E-08	0.160E-11	0.193E-20
15.9	0.332E-01	0.370E-06	0.647E-09	0.403E-12	0.165E-21
16.1	0.296E-01	0.188E-06	0.235E-09	0.102E-12	0.141E-22
16.3	0.264E-01	0.959E-07	0.851E-10	0.258E-13	0.120E-23
16.5	0.235E-01	0.489E-07	0.309E-10	0.653E-14	0.102E-24
16.7	0.210E-01	0.250E-07	0.112E-10	0.166E-14	0.872E-26
16.9	0.188E-01	0.128E-07	0.407E-11	0.421E-15	0.744E-27
17.1	0.168E-01	0.660E-08	0.148E-11	0.107E-15	0.634E-28
17.2	0.151E-01	0.338E-08	0.536E-12	0.272E-16	0.540E-29
17.4	0.136E-01	0.185E-08	0.195E-12	0.692E-17	0.461E-30
17.6	0.122E-01	0.985E-09	0.707E-13	0.176E-17	0.393E-31
17.7	0.109E-01	0.507E-09	0.257E-13	0.449E-18	0.335E-32
17.9	0.983E-02	0.359E-09	0.933E-14	0.114E-18	0.285E-33
18.0	0.885E-02	0.186E-09	0.339E-14	0.292E-19	0.243E-34
18.1	0.797E-02	0.965E-10	0.123E-14	0.745E-20	0.207E-35
18.3	0.718E-02	0.501E-10	0.448E-15	0.190E-20	0.176E-36
18.4	0.647E-02	0.260E-10	0.163E-15	0.486E-21	0.150E-37

* S/N in dB
* t/T stands for tau/T
* d stands for delta

TABLE(4.20):Pe vs. S/N due to Frequency-
Selective Fading for 4-QAM Modulation
with tau/T = 0.4 and delta variable

S/N	d = -0.7	d = -0.2	d = 0.0	d = 0.2	d = 0.7
0.0	0.494E 00	0.338E 00	0.292E 00	0.247E 00	0.164E 00
4.8	0.336E 00	0.127E 00	0.815E-01	0.538E-01	0.256E-01
7.0	0.252E 00	0.520E-01	0.252E-01	0.136E-01	0.546E-02
8.5	0.206E 00	0.226E-01	0.813E-02	0.371E-02	0.128E-02
9.5	0.180E 00	0.102E-01	0.270E-02	0.106E-02	0.313E-03
10.4	0.155E 00	0.471E-02	0.911E-03	0.315E-03	0.781E-04
11.1	0.136E 00	0.223E-02	0.311E-03	0.961E-04	0.198E-04
11.8	0.119E 00	0.107E-02	0.108E-03	0.298E-04	0.505E-05
12.3	0.105E 00	0.524E-03	0.374E-04	0.939E-05	0.130E-05
12.8	0.933E-01	0.258E-03	0.131E-04	0.299E-05	0.336E-06
13.2	0.829E-01	0.129E-03	0.459E-05	0.959E-06	0.875E-07
13.6	0.737E-01	0.644E-04	0.162E-05	0.310E-06	0.229E-07
14.0	0.657E-01	0.325E-04	0.573E-06	0.101E-06	0.600E-08
14.3	0.586E-01	0.164E-04	0.203E-06	0.329E-07	0.158E-08
14.6	0.524E-01	0.835E-05	0.724E-07	0.108E-07	0.437E-09
14.9	0.469E-01	0.426E-05	0.258E-07	0.353E-08	0.168E-09
15.2	0.420E-01	0.217E-05	0.923E-08	0.122E-08	0.451E-10
15.4	0.376E-01	0.111E-05	0.330E-08	0.579E-09	0.121E-10
15.7	0.337E-01	0.571E-06	0.124E-08	0.193E-09	0.323E-11
15.9	0.303E-01	0.294E-06	0.647E-09	0.645E-10	0.866E-12
16.1	0.272E-01	0.151E-06	0.235E-09	0.215E-10	0.232E-12
16.3	0.244E-01	0.780E-07	0.851E-10	0.719E-11	0.622E-13
16.5	0.220E-01	0.402E-07	0.309E-10	0.240E-11	0.167E-13
16.7	0.198E-01	0.208E-07	0.112E-10	0.802E-12	0.448E-14
16.9	0.178E-01	0.107E-07	0.407E-11	0.268E-12	0.120E-14
17.1	0.160E-01	0.556E-08	0.148E-11	0.897E-13	0.322E-15
17.2	0.144E-01	0.288E-08	0.536E-12	0.300E-13	0.865E-16
17.4	0.130E-01	0.149E-08	0.195E-12	0.100E-13	0.232E-16
17.6	0.117E-01	0.811E-09	0.707E-13	0.335E-14	0.623E-17
17.7	0.106E-01	0.422E-09	0.257E-13	0.112E-14	0.167E-17
17.9	0.954E-02	0.317E-09	0.933E-14	0.375E-15	0.449E-18
18.0	0.861E-02	0.166E-09	0.339E-14	0.126E-15	0.121E-18
18.1	0.777E-02	0.865E-10	0.123E-14	0.420E-16	0.324E-19
18.3	0.702E-02	0.452E-10	0.448E-15	0.141E-16	0.869E-20
18.4	0.634E-02	0.236E-10	0.163E-15	0.471E-17	0.233E-20

* S/N in dB
* t/T stands for tau/T
* d stands for delta

TABLE(4.21):Pe vs. S/N due to Frequency-
Selective Fading for 4-QAM Modulation
with tau/T = 0.7 and delta variable

S/N	d = -0.7	d = -0.2	d = 0.0	d = 0.2	d = 0.7
0.0	0.424E 00	0.315E 00	0.292E 00	0.272E 00	0.254E 00
4.8	0.290E 00	0.110E 00	0.815E-01	0.730E-01	0.105E 00
7.0	0.230E 00	0.442E-01	0.252E-01	0.233E-01	0.531E-01
8.5	0.196E 00	0.192E-01	0.813E-02	0.820E-02	0.282E-01
9.5	0.176E 00	0.878E-02	0.270E-02	0.305E-02	0.153E-01
10.4	0.153E 00	0.415E-02	0.911E-03	0.117E-02	0.845E-02
11.1	0.135E 00	0.201E-02	0.311E-03	0.462E-03	0.471E-02
11.8	0.119E 00	0.987E-03	0.108E-03	0.185E-03	0.265E-02
12.3	0.105E 00	0.490E-03	0.374E-04	0.747E-04	0.150E-02
12.8	0.932E-01	0.245E-03	0.131E-04	0.304E-04	0.849E-03
13.2	0.828E-01	0.124E-03	0.459E-05	0.125E-04	0.484E-03
13.6	0.737E-01	0.625E-04	0.162E-05	0.512E-05	0.277E-03
14.0	0.657E-01	0.317E-04	0.573E-06	0.211E-05	0.159E-03
14.3	0.586E-01	0.161E-04	0.203E-06	0.875E-06	0.916E-04
14.6	0.524E-01	0.824E-05	0.724E-07	0.363E-06	0.528E-04
14.9	0.469E-01	0.421E-05	0.258E-07	0.151E-06	0.305E-04
15.2	0.420E-01	0.216E-05	0.923E-08	0.629E-07	0.177E-04
15.4	0.376E-01	0.111E-05	0.330E-08	0.262E-07	0.102E-04
15.7	0.337E-01	0.569E-06	0.124E-08	0.110E-07	0.595E-05
15.9	0.303E-01	0.293E-06	0.647E-09	0.459E-08	0.346E-05
16.1	0.272E-01	0.151E-06	0.235E-09	0.192E-08	0.201E-05
16.3	0.244E-01	0.777E-07	0.851E-10	0.847E-09	0.117E-05
16.5	0.220E-01	0.401E-07	0.309E-10	0.514E-09	0.683E-06
16.7	0.198E-01	0.207E-07	0.112E-10	0.218E-09	0.399E-06
16.9	0.178E-01	0.107E-07	0.407E-11	0.922E-10	0.233E-06
17.1	0.160E-01	0.555E-08	0.148E-11	0.391E-10	0.136E-06
17.2	0.144E-01	0.287E-08	0.536E-12	0.166E-10	0.796E-07
17.4	0.130E-01	0.149E-08	0.195E-12	0.703E-11	0.466E-07
17.6	0.117E-01	0.811E-09	0.707E-13	0.298E-11	0.273E-07
17.7	0.106E-01	0.422E-09	0.257E-13	0.126E-11	0.160E-07
17.9	0.954E-02	0.317E-09	0.933E-14	0.536E-12	0.937E-08
18.0	0.861E-02	0.166E-09	0.339E-14	0.227E-12	0.550E-08
18.1	0.777E-02	0.865E-10	0.123E-14	0.965E-13	0.323E-08
18.3	0.702E-02	0.452E-10	0.448E-15	0.410E-13	0.189E-08
18.4	0.634E-02	0.236E-10	0.163E-15	0.174E-13	0.117E-08

* S/N in dB
* t/T stands for tau/T
* d stands for delta

TABLE(4.22):Pe vs. S/N due to Frequency-
Selective Fading for 4-QAM Modulation
with delta = 1.0 and tau/T variable

S/N	d = -0.7	d = -0.2	d = 0.0	d = 0.2	d = 0.7
0.0	0.381E 00	0.294E 00	0.292E 00	0.294E 00	0.381E 00
4.8	0.280E 00	0.992E-01	0.815E-01	0.992E-01	0.280E 00
7.0	0.228E 00	0.401E-01	0.252E-01	0.401E-01	0.228E 00
8.5	0.196E 00	0.178E-01	0.813E-02	0.178E-01	0.196E 00
9.5	0.176E 00	0.834E-02	0.270E-02	0.834E-02	0.176E 00
10.4	0.153E 00	0.402E-02	0.911E-03	0.402E-02	0.153E 00
11.1	0.135E 00	0.197E-02	0.311E-03	0.197E-02	0.135E 00
11.8	0.119E 00	0.974E-03	0.108E-03	0.974E-03	0.119E 00
12.3	0.105E 00	0.486E-03	0.374E-04	0.486E-03	0.105E 00
12.8	0.932E-01	0.244E-03	0.131E-04	0.244E-03	0.932E-01
13.2	0.828E-01	0.123E-03	0.459E-05	0.123E-03	0.828E-01
13.6	0.737E-01	0.624E-04	0.162E-05	0.624E-04	0.737E-01
14.0	0.657E-01	0.317E-04	0.573E-06	0.317E-04	0.657E-01
14.3	0.586E-01	0.161E-04	0.203E-06	0.161E-04	0.586E-01
14.6	0.524E-01	0.823E-05	0.724E-07	0.823E-05	0.524E-01
14.9	0.469E-01	0.421E-05	0.258E-07	0.421E-05	0.469E-01
15.2	0.420E-01	0.216E-05	0.923E-08	0.216E-05	0.420E-01
15.4	0.376E-01	0.111E-05	0.330E-08	0.111E-05	0.376E-01
15.7	0.337E-01	0.569E-06	0.124E-08	0.569E-06	0.337E-01
15.9	0.303E-01	0.293E-06	0.647E-09	0.293E-06	0.303E-01
16.1	0.272E-01	0.151E-06	0.235E-09	0.151E-06	0.272E-01
16.3	0.244E-01	0.777E-07	0.851E-10	0.777E-07	0.244E-01
16.5	0.220E-01	0.401E-07	0.309E-10	0.401E-07	0.220E-01
16.7	0.198E-01	0.207E-07	0.112E-10	0.207E-07	0.198E-01
16.9	0.178E-01	0.107E-07	0.407E-11	0.107E-07	0.178E-01
17.1	0.160E-01	0.555E-08	0.148E-11	0.555E-08	0.160E-01
17.2	0.144E-01	0.287E-08	0.536E-12	0.287E-08	0.144E-01
17.4	0.130E-01	0.149E-08	0.195E-12	0.149E-08	0.130E-01
17.6	0.117E-01	0.811E-09	0.707E-13	0.811E-09	0.117E-01
17.7	0.106E-01	0.422E-09	0.257E-13	0.422E-09	0.106E-01
17.9	0.954E-02	0.317E-09	0.933E-14	0.317E-09	0.954E-02
18.0	0.861E-02	0.166E-09	0.339E-14	0.166E-09	0.861E-02
18.1	0.777E-02	0.865E-10	0.123E-14	0.865E-10	0.777E-02
18.3	0.702E-02	0.452E-10	0.448E-15	0.452E-10	0.702E-02
18.4	0.634E-02	0.236E-10	0.163E-15	0.236E-10	0.634E-02

* S/N in dB
* t/T stands for tau/T
* d stands for delta

TABLE(4.23):Pe vs. S/N due to Frequency-
Selective Fading for 16-QAM Modulation
with beta = 0.1 and tau/T variable

S/N	t/T = 0.0	t/T = 0.2	t/T = 0.5	t/T = 0.7	t/T = 1.0
0.0	0.996E 00	0.999E 00	0.100E 01	0.100E 01	0.998E 00
4.8	0.831E 00	0.856E 00	0.910E 00	0.884E 00	0.855E 00
7.0	0.660E 00	0.702E 00	0.781E 00	0.751E 00	0.694E 00
8.5	0.513E 00	0.557E 00	0.660E 00	0.616E 00	0.552E 00
9.5	0.398E 00	0.438E 00	0.554E 00	0.501E 00	0.437E 00
10.4	0.310E 00	0.344E 00	0.463E 00	0.406E 00	0.345E 00
11.1	0.242E 00	0.270E 00	0.386E 00	0.329E 00	0.274E 00
11.8	0.190E 00	0.212E 00	0.323E 00	0.266E 00	0.217E 00
12.3	0.150E 00	0.167E 00	0.271E 00	0.216E 00	0.173E 00
12.8	0.119E 00	0.131E 00	0.227E 00	0.175E 00	0.139E 00
13.2	0.945E-01	0.103E 00	0.191E 00	0.142E 00	0.111E 00
13.6	0.757E-01	0.817E-01	0.161E 00	0.116E 00	0.897E-01
14.0	0.608E-01	0.646E-01	0.137E 00	0.941E-01	0.725E-01
14.3	0.490E-01	0.511E-01	0.116E 00	0.767E-01	0.588E-01
14.6	0.396E-01	0.405E-01	0.984E-01	0.627E-01	0.478E-01
14.9	0.321E-01	0.322E-01	0.838E-01	0.512E-01	0.390E-01
15.2	0.261E-01	0.256E-01	0.715E-01	0.420E-01	0.320E-01
15.4	0.213E-01	0.204E-01	0.612E-01	0.344E-01	0.262E-01
15.7	0.175E-01	0.163E-01	0.524E-01	0.283E-01	0.216E-01
15.9	0.143E-01	0.130E-01	0.450E-01	0.232E-01	0.178E-01
16.1	0.118E-01	0.104E-01	0.387E-01	0.191E-01	0.147E-01
16.3	0.968E-02	0.830E-02	0.334E-01	0.158E-01	0.122E-01
16.5	0.798E-02	0.665E-02	0.288E-01	0.130E-01	0.101E-01
16.7	0.659E-02	0.534E-02	0.249E-01	0.107E-01	0.844E-02
16.9	0.545E-02	0.428E-02	0.216E-01	0.887E-02	0.704E-02
17.1	0.452E-02	0.344E-02	0.187E-01	0.734E-02	0.588E-02
17.2	0.375E-02	0.277E-02	0.163E-01	0.608E-02	0.493E-02
17.4	0.311E-02	0.223E-02	0.141E-01	0.505E-02	0.413E-02
17.6	0.259E-02	0.180E-02	0.123E-01	0.419E-02	0.347E-02
17.7	0.216E-02	0.145E-02	0.107E-01	0.348E-02	0.292E-02
17.9	0.180E-02	0.117E-02	0.937E-02	0.289E-02	0.246E-02

* S/N in dB
* t/T stands for tau/T

TABLE(4.24):Pe vs. S/N due to Frequency-
Selective Fading for 16-QAM Modulation
with beta = 0.5 and tau/T variable

S/N	t/T = 0.0	t/T = 0.2	t/T = 0.5	t/T = 0.7	t/T = 1.0
0.0	0.954E 00	0.991E 00	0.981E 00	0.996E 00	0.987E 00
4.8	0.748E 00	0.809E 00	0.992E 00	0.954E 00	0.829E 00
7.0	0.618E 00	0.626E 00	0.967E 00	0.865E 00	0.705E 00
8.5	0.533E 00	0.489E 00	0.942E 00	0.775E 00	0.615E 00
9.5	0.461E 00	0.382E 00	0.920E 00	0.696E 00	0.543E 00
10.4	0.414E 00	0.301E 00	0.904E 00	0.631E 00	0.494E 00
11.1	0.377E 00	0.240E 00	0.891E 00	0.569E 00	0.461E 00
11.8	0.357E 00	0.192E 00	0.882E 00	0.518E 00	0.431E 00
12.3	0.329E 00	0.156E 00	0.904E 00	0.474E 00	0.406E 00
12.8	0.305E 00	0.127E 00	0.897E 00	0.438E 00	0.385E 00
13.2	0.284E 00	0.104E 00	0.891E 00	0.404E 00	0.367E 00
13.6	0.265E 00	0.862E-01	0.886E 00	0.375E 00	0.352E 00
14.0	0.247E 00	0.715E-01	0.882E 00	0.348E 00	0.339E 00
14.3	0.231E 00	0.596E-01	0.878E 00	0.325E 00	0.328E 00
14.6	0.216E 00	0.499E-01	0.875E 00	0.304E 00	0.318E 00
14.9	0.202E 00	0.419E-01	0.872E 00	0.282E 00	0.309E 00
15.2	0.189E 00	0.353E-01	0.867E 00	0.265E 00	0.301E 00
15.4	0.177E 00	0.298E-01	0.865E 00	0.250E 00	0.294E 00
15.7	0.166E 00	0.252E-01	0.863E 00	0.236E 00	0.288E 00
15.9	0.156E 00	0.214E-01	0.861E 00	0.224E 00	0.282E 00
16.1	0.146E 00	0.182E-01	0.859E 00	0.212E 00	0.276E 00
16.3	0.137E 00	0.155E-01	0.863E 00	0.202E 00	0.271E 00
16.5	0.129E 00	0.132E-01	0.861E 00	0.192E 00	0.267E 00
16.7	0.121E 00	0.113E-01	0.860E 00	0.183E 00	0.263E 00
16.9	0.114E 00	0.962E-02	0.854E 00	0.175E 00	0.259E 00
17.1	0.107E 00	0.824E-02	0.853E 00	0.170E 00	0.255E 00
17.2	0.101E 00	0.706E-02	0.855E 00	0.163E 00	0.252E 00
17.4	0.949E-01	0.606E-02	0.854E 00	0.156E 00	0.249E 00
17.6	0.893E-01	0.520E-02	0.854E 00	0.150E 00	0.246E 00
17.7	0.840E-01	0.447E-02	0.853E 00	0.144E 00	0.243E 00
17.9	0.791E-01	0.385E-02	0.852E 00	0.139E 00	0.241E 00

* S/N in dB
* t/T stands for tau/T

TABLE(4.25):Pe vs. S/N due to Frequency-Selective Fading for 16-QAM Modulation with beta = 1.0 and tau/T variable

S/N	t/T = 0.0	t/T = 0.2	t/T = 0.5	t/T = 0.7	t/T = 1.0
0.0	0.923E 00	0.976E 00	0.959E 00	0.988E 00	0.964E 00
4.8	0.808E 00	0.767E 00	0.960E 00	0.992E 00	0.839E 00
7.0	0.773E 00	0.596E 00	0.942E 00	0.962E 00	0.829E 00
8.5	0.759E 00	0.480E 00	0.947E 00	0.932E 00	0.800E 00
9.5	0.754E 00	0.399E 00	0.949E 00	0.880E 00	0.782E 00
10.4	0.752E 00	0.340E 00	0.945E 00	0.857E 00	0.769E 00
11.1	0.751E 00	0.292E 00	0.946E 00	0.839E 00	0.766E 00
11.8	0.751E 00	0.253E 00	0.943E 00	0.824E 00	0.754E 00
12.3	0.751E 00	0.221E 00	0.890E 00	0.812E 00	0.749E 00
12.8	0.751E 00	0.195E 00	0.882E 00	0.838E 00	0.745E 00
13.2	0.751E 00	0.172E 00	0.901E 00	0.828E 00	0.741E 00
13.6	0.751E 00	0.153E 00	0.887E 00	0.820E 00	0.739E 00
14.0	0.751E 00	0.137E 00	0.885E 00	0.813E 00	0.737E 00
14.3	0.751E 00	0.122E 00	0.884E 00	0.806E 00	0.735E 00
14.6	0.751E 00	0.110E 00	0.883E 00	0.800E 00	0.733E 00
14.9	0.751E 00	0.987E-01	0.882E 00	0.795E 00	0.732E 00
15.2	0.751E 00	0.888E-01	0.881E 00	0.790E 00	0.731E 00
15.4	0.751E 00	0.802E-01	0.872E 00	0.786E 00	0.730E 00
15.7	0.751E 00	0.725E-01	0.891E 00	0.782E 00	0.729E 00
15.9	0.751E 00	0.656E-01	0.900E 00	0.778E 00	0.728E 00
16.1	0.751E 00	0.594E-01	0.878E 00	0.775E 00	0.727E 00
16.3	0.751E 00	0.539E-01	0.867E 00	0.771E 00	0.727E 00
16.5	0.751E 00	0.489E-01	0.867E 00	0.775E 00	0.726E 00
16.7	0.751E 00	0.445E-01	0.867E 00	0.770E 00	0.725E 00
16.9	0.751E 00	0.405E-01	0.875E 00	0.767E 00	0.725E 00
17.1	0.751E 00	0.368E-01	0.874E 00	0.759E 00	0.724E 00
17.2	0.751E 00	0.335E-01	0.873E 00	0.757E 00	0.724E 00
17.4	0.751E 00	0.306E-01	0.873E 00	0.754E 00	0.723E 00
17.6	0.751E 00	0.279E-01	0.872E 00	0.753E 00	0.723E 00
17.7	0.751E 00	0.255E-01	0.872E 00	0.751E 00	0.722E 00
17.9	0.751E 00	0.233E-01	0.871E 00	0.749E 00	0.722E 00

* S/N in dB

* t/T stands for tau/T

TABLE(4.26):16-QAM Pe variation with tau/T
for different S/N and beta = 0.1

t/T	S/N=10.4dB	S/N=13.2dB	S/N=17.9dB
0.00	0.30986E 00	0.94543E-01	0.17958E-02
0.04	0.38524E 00	0.12654E 00	0.17981E-02
0.08	0.48605E 00	0.21309E 00	0.13077E-01
0.12	0.33921E 00	0.10128E 00	0.11735E-02
0.16	0.31885E 00	0.95349E-01	0.14700E-02
0.20	0.44003E 00	0.16927E 00	0.54890E-02
0.24	0.43518E 00	0.16509E 00	0.49886E-02
0.28	0.32088E 00	0.96071E-01	0.14807E-02
0.32	0.34420E 00	0.10350E 00	0.11814E-02
0.36	0.46849E 00	0.19626E 00	0.99477E-02
0.40	0.37975E 00	0.12295E 00	0.16373E-02
0.44	0.32044E 00	0.97385E-01	0.17233E-02
0.48	0.38477E 00	0.12628E 00	0.17928E-02
0.52	0.45673E 00	0.18578E 00	0.83695E-02
0.56	0.34488E 00	0.10419E 00	0.12396E-02
0.60	0.33156E 00	0.10054E 00	0.15494E-02
0.64	0.42551E 00	0.15795E 00	0.44593E-02
0.68	0.41694E 00	0.15086E 00	0.37100E-02
0.72	0.33416E 00	0.10275E 00	0.17439E-02
0.76	0.35295E 00	0.10807E 00	0.12784E-02
0.80	0.44559E 00	0.17782E 00	0.77672E-02
0.84	0.37538E 00	0.12020E 00	0.15291E-02
0.88	0.33871E 00	0.10756E 00	0.22884E-02
0.92	0.38304E 00	0.12534E 00	0.17794E-02
0.96	0.43520E 00	0.16989E 00	0.68558E-02
1.00	0.35337E 00	0.10952E 00	0.14742E-02

* t/T stands for tau/T

TABLE(4.27):16-QAM Pe variation with tau/T for different S/N and beta = 0.5

t/T	S/N=10.4dB	S/N=13.2dB	S/N=17.9dB
0.00	0.41400E 00	0.28396E 00	0.79108E-01
0.04	0.45539E 00	0.18307E 00	0.73894E-02
0.08	0.93055E 00	0.94768E 00	0.99587E 00
0.12	0.30173E 00	0.11323E 00	0.54114E-02
0.16	0.36212E 00	0.20519E 00	0.36820E-01
0.20	0.85595E 00	0.75248E 00	0.60959E 00
0.24	0.82754E 00	0.70363E 00	0.52151E 00
0.28	0.36153E 00	0.20594E 00	0.44043E-01
0.32	0.30369E 00	0.10766E 00	0.46259E-02
0.36	0.92491E 00	0.89621E 00	0.90920E 00
0.40	0.41990E 00	0.15246E 00	0.36917E-02
0.44	0.39714E 00	0.25738E 00	0.10248E 00
0.48	0.45393E 00	0.18285E 00	0.77677E-02
0.52	0.87903E 00	0.82072E 00	0.80105E 00
0.56	0.31665E 00	0.12141E 00	0.85451E-02
0.60	0.37152E 00	0.20801E 00	0.65552E-01
0.64	0.77783E 00	0.62671E 00	0.43344E 00
0.68	0.71364E 00	0.53433E 00	0.29727E 00
0.72	0.40148E 00	0.24950E 00	0.10620E 00
0.76	0.32775E 00	0.11703E 00	0.59781E-02
0.80	0.86725E 00	0.83831E 00	0.72418E 00
0.84	0.39452E 00	0.13355E 00	0.22838E-02
0.88	0.46969E 00	0.34251E 00	0.22096E 00
0.92	0.44969E 00	0.18369E 00	0.94115E-02
0.96	0.82587E 00	0.74658E 00	0.71546E 00
1.00	0.36584E 00	0.16602E 00	0.22676E-01

* t/T stands for tau/T

TABLE(4.28):16-QAM Pe variation with tau/T for different S/N and beta = 1.0

t/T	S/N=10.4dB	S/N=13.2dB	S/N=17.9dB
0.00	0.75206E 00	0.75058E 00	0.75056E 00
0.04	0.56840E 00	0.31033E 00	0.47927E-01
0.08	0.79723E 00	0.80182E 00	0.84655E 00
0.12	0.36819E 00	0.21270E 00	0.38975E-01
0.16	0.60741E 00	0.54406E 00	0.41846E 00
0.20	0.97041E 00	0.97523E 00	0.98989E 00
0.24	0.99575E 00	0.98842E 00	0.99423E 00
0.28	0.59737E 00	0.47740E 00	0.38193E 00
0.32	0.34676E 00	0.18586E 00	0.34350E-01
0.36	0.89746E 00	0.89477E 00	0.87488E 00
0.40	0.48285E 00	0.21094E 00	0.13217E-01
0.44	0.59568E 00	0.62378E 00	0.60945E 00
0.48	0.56443E 00	0.31265E 00	0.54908E-01
0.52	0.96493E 00	0.93983E 00	0.89611E 00
0.56	0.38579E 00	0.23920E 00	0.92064E-01
0.60	0.51378E 00	0.44614E 00	0.48302E 00
0.64	0.99165E 00	0.98341E 00	0.97466E 00
0.68	0.97141E 00	0.95293E 00	0.95660E 00
0.72	0.56801E 00	0.58205E 00	0.54614E 00
0.76	0.37378E 00	0.19894E 00	0.56627E-01
0.80	0.98309E 00	0.98781E 00	0.98749E 00
0.84	0.42368E 00	0.15777E 00	0.47142E-02
0.88	0.73336E 00	0.70335E 00	0.66826E 00
0.90	0.91935E 00	0.87783E 00	0.87424E 00
0.94	0.70877E 00	0.67693E 00	0.62134E 00
0.98	0.40999E 00	0.22914E 00	0.74271E-01
1.00	0.49872E 00	0.37451E 00	0.25105E 00

* t/T stands for tau/T

TABLE(4.29):16-QAM Pe variation with beta
for different S/N and tau/T = 0.1

beta	S/N=10.4dB	S/N=13.2dB	S/N=17.9dB
0.00	0.37117E 00	0.11760E 00	0.14332E-02
0.04	0.35747E 00	0.10984E 00	0.12310E-02
0.08	0.34539E 00	0.10391E 00	0.11653E-02
0.12	0.33489E 00	0.99613E-01	0.11954E-02
0.16	0.32592E 00	0.96753E-01	0.12989E-02
0.20	0.31834E 00	0.95159E-01	0.14652E-02
0.24	0.31230E 00	0.94648E-01	0.16914E-02
0.28	0.30750E 00	0.95120E-01	0.19794E-02
0.32	0.30393E 00	0.96491E-01	0.23348E-02
0.36	0.30152E 00	0.98668E-01	0.27654E-02
0.42	0.29988E 00	0.10321E 00	0.35749E-02
0.46	0.30001E 00	0.10698E 00	0.42418E-02
0.50	0.30099E 00	0.11132E 00	0.50275E-02
0.54	0.30279E 00	0.11619E 00	0.59491E-02
0.58	0.30536E 00	0.12155E 00	0.70252E-02
0.62	0.30864E 00	0.12739E 00	0.82772E-02
0.66	0.31251E 00	0.13369E 00	0.97284E-02
0.70	0.31701E 00	0.14043E 00	0.11404E-01
0.74	0.32201E 00	0.14761E 00	0.13333E-01
0.78	0.32754E 00	0.15522E 00	0.15546E-01
0.82	0.33349E 00	0.16325E 00	0.18075E-01
0.86	0.33985E 00	0.17170E 00	0.20955E-01
0.90	0.34660E 00	0.18056E 00	0.24225E-01
0.94	0.35371E 00	0.18982E 00	0.27925E-01
1.00	0.36207E 00	0.20447E 00	0.34370E-01

TABLE(4.30):16-QAM Pe variation with beta
for different S/N and tau/T = 0.7

beta	S/N=10.4dB	S/N=13.2dB	S/N=17.9dB
0.00	0.37117E 00	0.11760E 00	0.14332E-02
0.04	0.37613E 00	0.12066E 00	0.15458E-02
0.08	0.38134E 00	0.12405E 00	0.16948E-02
0.12	0.38681E 00	0.12778E 00	0.18866E-02
0.16	0.39255E 00	0.13186E 00	0.21294E-02
0.20	0.39855E 00	0.13628E 00	0.24331E-02
0.24	0.40481E 00	0.14111E 00	0.28097E-02
0.28	0.41132E 00	0.14636E 00	0.32737E-02
0.32	0.41810E 00	0.15193E 00	0.38420E-02
0.36	0.42513E 00	0.15796E 00	0.45347E-02
0.40	0.43242E 00	0.16448E 00	0.53755E-02
0.44	0.43995E 00	0.17138E 00	0.63915E-02
0.48	0.44773E 00	0.17872E 00	0.76139E-02
0.52	0.45574E 00	0.18649E 00	0.90782E-02
0.56	0.46399E 00	0.19475E 00	0.10824E-01
0.60	0.47247E 00	0.20345E 00	0.12898E-01
0.64	0.48114E 00	0.21267E 00	0.15347E-01
0.68	0.49006E 00	0.22234E 00	0.18226E-01
0.72	0.49917E 00	0.23251E 00	0.21594E-01
0.76	0.50593E 00	0.24313E 00	0.25512E-01
0.80	0.51537E 00	0.25422E 00	0.30017E-01
0.84	0.52499E 00	0.26578E 00	0.35267E-01
0.88	0.53479E 00	0.27780E 00	0.41240E-01
0.92	0.54476E 00	0.29027E 00	0.48037E-01
0.96	0.55525E 00	0.30318E 00	0.55726E-01
1.00	0.56548E 00	0.31354E 00	0.64373E-01

TABLE(4.31):16-QAM Pe variation with beta
for different S/N and tau/T = 1.0

beta	S/N=10.4dB	S/N=13.2dB	S/N=17.9dB
0.00	0.37117E 00	0.11760E 00	0.14332E-02
0.04	0.36286E 00	0.11304E 00	0.13368E-02
0.08	0.35614E 00	0.11027E 00	0.13927E-02
0.12	0.35099E 00	0.10914E 00	0.15925E-02
0.16	0.34734E 00	0.10961E 00	0.19519E-02
0.20	0.34520E 00	0.11151E 00	0.25079E-02
0.24	0.34440E 00	0.11472E 00	0.33183E-02
0.28	0.34503E 00	0.11936E 00	0.44632E-02
0.32	0.34688E 00	0.12519E 00	0.60464E-02
0.36	0.34992E 00	0.13225E 00	0.81976E-02
0.40	0.35410E 00	0.14048E 00	0.11073E-01
0.44	0.35925E 00	0.14986E 00	0.14854E-01
0.48	0.36246E 00	0.16036E 00	0.19745E-01
0.52	0.36952E 00	0.17194E 00	0.25968E-01
0.56	0.37739E 00	0.18457E 00	0.33749E-01
0.60	0.38931E 00	0.19820E 00	0.43311E-01
0.64	0.39872E 00	0.20939E 00	0.54853E-01
0.68	0.40877E 00	0.22470E 00	0.68539E-01
0.72	0.41937E 00	0.24455E 00	0.84474E-01
0.76	0.43048E 00	0.26159E 00	0.10269E 00
0.80	0.44202E 00	0.27930E 00	0.11956E 00
0.84	0.45393E 00	0.29759E 00	0.14568E 00
0.88	0.46616E 00	0.31635E 00	0.17008E 00
0.92	0.47336E 00	0.33550E 00	0.19603E 00
0.96	0.48596E 00	0.35492E 00	0.22316E 00
1.00	0.49872E 00	0.37451E 00	0.25105E 00

TABLE(5.1):Peak Distortion variation with the relative delay for the Two-Ray Model,with a 5-taps equalizers and beta = 0.1

t/T	No Equalizer	Z.F Equalizer	MMSE Equalizer
0.00	0.00000E 00	0.00000E 00	0.00000E 00
0.06	0.10010E 00	0.10433E-02	0.11396E-02
0.13	0.10036E 00	0.10143E-01	0.11046E-01
0.19	0.10050E 00	0.10029E 00	0.10033E 00
0.25	0.10085E 00	0.10062E 00	0.10069E 00
0.31	0.10099E 00	0.10078E 00	0.10088E 00
0.38	0.10125E 00	0.10108E 00	0.10121E 00
0.44	0.10139E 00	0.10121E 00	0.10133E 00
0.50	0.10180E 00	0.10153E 00	0.10162E 00
0.56	0.10193E 00	0.10164E 00	0.10174E 00
0.63	0.10218E 00	0.10184E 00	0.10201E 00
0.69	0.10231E 00	0.10185E 00	0.10207E 00
0.75	0.10263E 00	0.10194E 00	0.10217E 00
0.81	0.10273E 00	0.10162E 00	0.10180E 00
0.88	0.10290E 00	0.11172E-01	0.12090E-01
0.94	0.10275E 00	0.23661E-02	0.24815E-02
1.00	0.33836E-02	0.16031E-02	0.16031E-02
1.06	0.10311E 00	0.26484E-02	0.27657E-02
1.13	0.10371E 00	0.11544E-01	0.12465E-01
1.19	0.10397E 00	0.10237E 00	0.10264E 00

* t/T stands tau/T
* Z.F = Zero Forcing
* MMSE = Minimum Mean Square Error

TABLE(5.2):Mean Square Distortion variation with the relative delay for the Two-Ray Model,with a 5-taps equalizers and beta = 0.5

t/T	No Equalizer	Z.F Equalizer	MMSE Equalizer
0.00	0.00000E 00	0.00000E 00	0.00000E 00
0.06	0.24990E 00	0.15553E-01	0.11809E-01
0.13	0.24990E 00	0.62346E-01	0.49268E-01
0.19	0.24991E 00	0.24991E 00	0.24991E 00
0.25	0.24992E 00	0.24992E 00	0.24992E 00
0.31	0.24993E 00	0.24993E 00	0.24993E 00
0.38	0.24995E 00	0.24995E 00	0.24994E 00
0.44	0.24997E 00	0.24997E 00	0.24997E 00
0.50	0.25000E 00	0.25000E 00	0.25000E 00
0.56	0.25003E 00	0.25003E 00	0.25003E 00
0.63	0.25008E 00	0.25007E 00	0.25007E 00
0.69	0.25014E 00	0.25013E 00	0.25013E 00
0.75	0.25023E 00	0.25022E 00	0.25021E 00
0.81	0.25038E 00	0.25036E 00	0.25035E 00
0.88	0.25066E 00	0.62166E-01	0.49093E-01
0.94	0.25149E 00	0.15551E-01	0.11798E-01
1.00	0.14305E-04	0.19073E-05	0.28610E-05
1.06	0.24831E 00	0.14920E-01	0.11364E-01
1.13	0.24914E 00	0.61215E-01	0.48414E-01
1.19	0.24943E 00	0.24939E 00	0.24938E 00

* t/T stands tau/T
* Z.F = Zero Forcing
* MMSE = Minimum Mean Square Error

TABLE(5.3):MMSE or ISI Distortion variation with the relative delay for the Two-Ray Model,with a 5-taps equalizers and beta = 1.0

t/T	No Equalizer	Z.F Equalizer	MMSE Equalizer
0.00	0.18778E 02	0.35527E-14	0.00000E 00
0.06	0.37805E 01	0.99498E 00	0.22753E 00
0.13	0.37804E 01	0.99731E 00	0.39545E 00
0.19	0.37802E 01	0.99929E 00	0.12491E 01
0.25	0.37799E 01	0.99936E 00	0.12492E 01
0.31	0.37796E 01	0.99946E 00	0.12493E 01
0.38	0.37791E 01	0.99958E 00	0.12495E 01
0.44	0.37786E 01	0.99976E 00	0.12497E 01
0.50	0.37778E 01	0.99998E 00	0.12500E 01
0.56	0.37774E 01	0.99979E 00	0.12498E 01
0.63	0.37770E 01	0.99949E 00	0.12495E 01
0.69	0.37765E 01	0.99902E 00	0.12490E 01
0.75	0.37757E 01	0.99831E 00	0.12483E 01
0.81	0.37745E 01	0.99715E 00	0.12472E 01
0.88	0.37722E 01	0.99574E 00	0.39693E 00
0.94	0.37656E 01	0.98204E 00	0.22896E 00
1.00	0.18774E 02	0.38147E-05	0.66758E-05
1.06	0.38245E 01	0.94867E 00	0.22507E 00
1.13	0.38016E 01	0.98265E 00	0.39332E 00
1.19	0.37935E 01	0.99531E 00	0.12440E 01

* t/T stands tau/T
* Z.F = Zero Forcing
* MMSE = Minimum Mean Square Error

TABLE(5.4):Peak Distortion variation with the relative delay for the Three-Ray Model,with a 5-taps equalizers and beta = 0.1

t/T	No Equalizer	Z.F Equalizer	MMSE Equalizer
0.00	0.00000E 00	0.00000E 00	0.00000E 00
0.04	0.21532E 00	0.72639E-01	0.75080E-01
0.08	0.20665E 00	0.57694E-01	0.59612E-01
0.12	0.15396E 00	0.31554E-01	0.32535E-01
0.16	0.25784E 00	0.14235E 00	0.15109E 00
0.20	0.17781E 00	0.14703E 00	0.15313E 00
0.24	0.20366E 00	0.17203E 00	0.18182E 00
0.28	0.25770E 00	0.21550E 00	0.23221E 00
0.32	0.12711E 00	0.12138E 00	0.12404E 00
0.35	0.24014E 00	0.21054E 00	0.22595E 00
0.39	0.23812E 00	0.21125E 00	0.22622E 00
0.43	0.13099E 00	0.12589E 00	0.12851E 00
0.47	0.26054E 00	0.23255E 00	0.24701E 00
0.51	0.20235E 00	0.18566E 00	0.19174E 00
0.55	0.18348E 00	0.16993E 00	0.17477E 00
0.59	0.26390E 00	0.23388E 00	0.24880E 00
0.63	0.15351E 00	0.14389E 00	0.14862E 00
0.67	0.22432E 00	0.19723E 00	0.21046E 00
0.71	0.24954E 00	0.21208E 00	0.22814E 00
0.75	0.10343E 00	0.10254E 00	0.10283E 00
0.79	0.24887E 00	0.19327E 00	0.20656E 00
0.83	0.21798E 00	0.15330E 00	0.16104E 00
0.87	0.15597E 00	0.34978E-01	0.36116E-01
0.91	0.24886E 00	0.79104E-01	0.82479E-01
0.95	0.16594E 00	0.36781E-01	0.37677E-01
0.99	0.11101E 00	0.49026E-01	0.50031E-01
1.03	0.18701E 00	0.74937E-01	0.77300E-01
1.06	0.12022E 00	0.11828E-01	0.12067E-01
1.10	0.23275E 00	0.68179E-01	0.71090E-01
1.14	0.22901E 00	0.90439E-01	0.95470E-01

* t/T stands tau/T
* Z.F = Zero Forcing
* MMSE = Minimum Mean Square Error

TABLE(5.5):Mean Square Distortion variation with the relative delay for the Three-Ray Model,with a 5-taps equalizers and beta = 0.5

t/T	No Equalizer	Z.F Equalizer	MMSE Equalizer
0.00	0.00000E 00	0.00000E 00	0.00000E 00
0.04	0.21300E 00	0.71917E-02	0.65670E-02
0.08	0.20699E 00	0.26283E-02	0.25311E-02
0.12	0.26016E 00	0.53047E-01	0.41965E-01
0.16	0.23238E 00	0.82399E-01	0.76569E-01
0.20	0.23727E 00	0.22261E 00	0.21828E 00
0.24	0.25126E 00	0.22947E 00	0.21732E 00
0.28	0.23152E 00	0.18881E 00	0.15876E 00
0.32	0.24801E 00	0.24699E 00	0.24607E 00
0.35	0.24158E 00	0.21308E 00	0.18509E 00
0.39	0.23650E 00	0.21186E 00	0.18540E 00
0.43	0.25004E 00	0.24912E 00	0.24803E 00
0.47	0.23523E 00	0.21070E 00	0.18760E 00
0.51	0.24429E 00	0.23762E 00	0.23445E 00
0.55	0.24496E 00	0.24058E 00	0.23857E 00
0.59	0.23460E 00	0.20733E 00	0.18241E 00
0.63	0.25038E 00	0.24726E 00	0.24386E 00
0.67	0.23647E 00	0.21436E 00	0.19436E 00
0.71	0.24044E 00	0.20271E 00	0.17390E 00
0.75	0.24969E 00	0.24967E 00	0.24966E 00
0.79	0.22862E 00	0.17782E 00	0.15965E 00
0.83	0.25257E 00	0.19619E 00	0.18853E 00
0.87	0.23645E 00	0.41859E-01	0.34089E-01
0.91	0.21921E 00	0.67854E-02	0.63429E-02
0.95	0.27894E 00	0.13645E-01	0.10559E-01
0.99	0.18856E-01	0.23746E-02	0.23432E-02
1.03	0.80054E-01	0.51670E-02	0.49810E-02
1.06	0.23938E 00	0.11527E-01	0.89569E-02
1.10	0.25056E 00	0.21125E-01	0.17577E-01
1.14	0.22298E 00	0.26541E-01	0.24412E-01

* t/T stands tau/T
* Z.F = Zero Forcing
* MMSE = Minimum Mean Square Error

TABLE(5.6):MMSE or ISI Distortion variation with the relative delay for the Three-Ray Model,with a 5-taps equalizers and beta = 1.0

t/T	No Equalizer	Z.F Equalizer	MMSE Equalizer
0.00	0.18778E 02	0.35527E-14	0.00000E 00
0.04	0.33878E 01	0.20169E 00	0.10111E 00
0.08	0.51119E 01	0.42645E-01	0.25736E-01
0.12	0.36032E 01	0.99752E 00	0.41070E 00
0.16	0.42103E 01	0.34794E 00	0.31428E 00
0.20	0.41473E 01	0.84159E 00	0.99920E 00
0.24	0.37235E 01	0.89475E 00	0.10123E 01
0.28	0.43194E 01	0.64035E 00	0.54381E 00
0.32	0.38326E 01	0.97797E 00	0.12148E 01
0.35	0.40026E 01	0.76735E 00	0.72920E 00
0.39	0.41699E 01	0.75423E 00	0.72174E 00
0.43	0.37757E 01	0.99462E 00	0.12331E 01
0.47	0.42079E 01	0.75189E 00	0.73502E 00
0.51	0.39330E 01	0.92024E 00	0.11300E 01
0.55	0.39159E 01	0.93758E 00	0.11593E 01
0.59	0.42260E 01	0.73414E 00	0.69153E 00
0.63	0.37646E 01	0.98401E 00	0.11988E 01
0.67	0.41713E 01	0.77236E 00	0.80115E 00
0.71	0.40268E 01	0.71751E 00	0.65357E 00
0.75	0.37863E 01	0.99742E 00	0.12467E 01
0.79	0.44066E 01	0.61549E 00	0.57308E 00
0.83	0.36620E 01	0.79797E 00	0.86305E 00
0.87	0.41725E 01	0.71139E 00	0.33911E 00
0.91	0.44710E 01	0.11118E 00	0.76883E-01
0.95	0.33219E 01	0.68665E 00	0.24753E 00
0.99	0.14990E 02	0.44708E-02	0.43713E-02
1.03	0.82645E 01	0.99821E-02	0.94025E-02
1.06	0.40835E 01	0.70662E 00	0.20589E 00
1.10	0.35973E 01	0.59477E 00	0.29533E 00
1.14	0.45866E 01	0.32336E 00	0.21012E 00

* t/T stands tau/T
* Z.F = Zero Forcing
* MMSE = Minimum Mean Square Error

TABLE(5.7):Peak Distortion variation with the component Ao for the Polynomial Model,with a 5-taps equalizers , B1 = 0.01*Ao and A1 = 0.001*Ao

Ao	No Equalizer	Z.F Equalizer	MMSE Equalizer
0.004	0.16859E 00	0.73902E-01	0.76080E-01
0.005	0.16860E 00	0.73903E-01	0.76080E-01
0.007	0.16860E 00	0.73902E-01	0.76081E-01
0.008	0.16860E 00	0.73902E-01	0.76080E-01
0.010	0.16860E 00	0.73903E-01	0.76079E-01
0.013	0.16860E 00	0.73902E-01	0.76080E-01
0.016	0.16860E 00	0.73903E-01	0.76080E-01
0.020	0.16860E 00	0.73903E-01	0.76080E-01
0.025	0.16859E 00	0.73902E-01	0.76081E-01
0.032	0.16859E 00	0.73901E-01	0.76081E-01
0.040	0.16859E 00	0.73903E-01	0.76079E-01
0.050	0.16859E 00	0.73902E-01	0.76079E-01
0.063	0.16859E 00	0.73903E-01	0.76080E-01
0.079	0.16860E 00	0.73903E-01	0.76080E-01
0.099	0.16860E 00	0.73903E-01	0.76081E-01
0.124	0.16860E 00	0.73903E-01	0.76081E-01
0.156	0.16860E 00	0.73902E-01	0.76080E-01
0.196	0.16860E 00	0.73903E-01	0.76080E-01
0.245	0.16860E 00	0.73903E-01	0.76080E-01
0.308	0.16860E 00	0.73903E-01	0.76080E-01
0.386	0.16859E 00	0.73902E-01	0.76080E-01
0.484	0.16859E 00	0.73902E-01	0.76081E-01
0.608	0.16859E 00	0.73902E-01	0.76079E-01
0.762	0.16859E 00	0.73902E-01	0.76079E-01
0.956	0.16859E 00	0.73903E-01	0.76080E-01
1.199	0.16860E 00	0.73902E-01	0.76080E-01
1.504	0.16860E 00	0.73903E-01	0.76080E-01
1.887	0.16860E 00	0.73903E-01	0.76081E-01
2.367	0.16860E 00	0.73903E-01	0.76079E-01
2.969	0.16860E 00	0.73903E-01	0.76079E-01

* t/T stands tau/T
* Z.F = Zero Forcing
* MMSE = Minimum Mean Square Error

TABLE(5.8):Mean Square Distortion variation with the component Ao for the Polynomial Model,with a 5-taps equalizers , B1 = 0.01*Ao and A1 = 0.001*Ao

Ao	No Equalizer	Z.F Equalizer	MMSE Equalizer
0.004	0.27733E-02	0.50449E-03	0.50545E-03
0.005	0.27733E-02	0.50449E-03	0.50545E-03
0.007	0.27704E-02	0.50449E-03	0.50545E-03
0.008	0.27704E-02	0.50449E-03	0.50545E-03
0.010	0.27723E-02	0.50449E-03	0.50545E-03
0.013	0.27723E-02	0.50449E-03	0.50545E-03
0.016	0.27733E-02	0.50449E-03	0.50545E-03
0.020	0.27733E-02	0.50449E-03	0.50545E-03
0.025	0.27695E-02	0.50449E-03	0.50545E-03
0.032	0.27704E-02	0.50449E-03	0.50545E-03
0.040	0.27723E-02	0.50449E-03	0.50545E-03
0.050	0.27723E-02	0.50449E-03	0.50545E-03
0.063	0.27733E-02	0.50449E-03	0.50545E-03
0.079	0.27733E-02	0.50449E-03	0.50545E-03
0.099	0.27733E-02	0.50449E-03	0.50545E-03
0.124	0.27704E-02	0.50449E-03	0.50545E-03
0.156	0.27723E-02	0.50449E-03	0.50545E-03
0.196	0.27723E-02	0.50449E-03	0.50545E-03
0.245	0.27733E-02	0.50449E-03	0.50545E-03
0.308	0.27733E-02	0.50449E-03	0.50545E-03
0.386	0.27733E-02	0.50449E-03	0.50545E-03
0.484	0.27704E-02	0.50449E-03	0.50545E-03
0.608	0.27723E-02	0.50449E-03	0.50545E-03
0.762	0.27733E-02	0.50449E-03	0.50545E-03
0.956	0.27733E-02	0.50449E-03	0.50545E-03
1.199	0.27733E-02	0.50449E-03	0.50545E-03
1.504	0.27733E-02	0.50449E-03	0.50545E-03
1.887	0.27685E-02	0.50449E-03	0.50545E-03
2.367	0.27714E-02	0.50449E-03	0.50545E-03
2.969	0.27723E-02	0.50449E-03	0.50545E-03

* t/T stands tau/T
* Z.F = Zero Forcing
* MMSE = Minimum Mean Square Error

TABLE(5.9):MMSE or ISI Distortion variation with the component Ao for the Polynomial Model,with a 5-taps equalizers , B1 = 0.01*Ao and A1 = 0.001*Ao

Ao	No Equalizer	Z.F Equalizer	MMSE Equalizer
0.004	0.98248E 00	0.50449E-03	0.50570E-03
0.005	0.97735E 00	0.50449E-03	0.50571E-03
0.007	0.97093E 00	0.50449E-03	0.50570E-03
0.008	0.96292E 00	0.50449E-03	0.50570E-03
0.010	0.95291E 00	0.50449E-03	0.50570E-03
0.013	0.94043E 00	0.50449E-03	0.50570E-03
0.016	0.92489E 00	0.50449E-03	0.50570E-03
0.020	0.90558E 00	0.50449E-03	0.50571E-03
0.025	0.88165E 00	0.50449E-03	0.50570E-03
0.032	0.85208E 00	0.50449E-03	0.50570E-03
0.040	0.81572E 00	0.50449E-03	0.50570E-03
0.050	0.77122E 00	0.50449E-03	0.50570E-03
0.063	0.71717E 00	0.50449E-03	0.50570E-03
0.079	0.65217E 00	0.50449E-03	0.50571E-03
0.099	0.57501E 00	0.50449E-03	0.50571E-03
0.124	0.48512E 00	0.50449E-03	0.50570E-03
0.156	0.38321E 00	0.50449E-03	0.50570E-03
0.196	0.27247E 00	0.50449E-03	0.50570E-03
0.245	0.16042E 00	0.50449E-03	0.50571E-03
0.308	0.62144E-01	0.50449E-03	0.50571E-03
0.386	0.54011E-02	0.50449E-03	0.50571E-03
0.484	0.38901E-01	0.50449E-03	0.50570E-03
0.608	0.24564E 00	0.50449E-03	0.50570E-03
0.762	0.76414E 00	0.50449E-03	0.50570E-03
0.956	0.18223E 01	0.50449E-03	0.50570E-03
1.199	0.37913E 01	0.50449E-03	0.50570E-03
1.504	0.72709E 01	0.50449E-03	0.50571E-03
1.887	0.13224E 02	0.50449E-03	0.50570E-03
2.367	0.23192E 02	0.50449E-03	0.50570E-03
2.969	0.39628E 02	0.50449E-03	0.50570E-03

* t/T stands tau/T
* Z.F = Zero Forcing
* MMSE = Minimum Mean Square Error

APPENDIX-VI

TABLE I :M-PSK and M-QAM performances comparison

Carrier Frequency : 4 GHz

Available Transmission BW : 20 MHz

Roll-off Factor : 0.5

S/N at the Receiver i/p : 20 dB

M S	Pe	Th S E	P T R	V S C
8-PSK	0.436E-06	3	40	625
16-PSK	0.222E-01	4	53	833
16-QAM	0.116E-04	4	53	833
64-QAM	0.502E-01	6	80	1250

* M S : Modulation Scheme
* Th S E : Theoritical Spectral Efficiency (b/s/Hz)
* P T R : Practical Transmission Rate (Mb/s)
* V S C : Voice Signal Capacity (voice signals)

TABLE II :Frequency-Selective Fading Effects On 16-QAM

Carrier Frequency : 4 GHz

Available Transmission BW : 20 MHz

Roll-off Factor : 0.5

S/N at the Receiver i/p : 20 dB

Transmission Rate : 53 Mb/s

Tau/T	Pe	Variation
0.00	0.57E-04	----
0.22	0.19E-04	-67%
0.52	0.80E-03	+1200%

* Tau : secondary-ray relative delay

APPENDIX-VII

```
C PROGRAM USED TO ESTIMATE THE OUTAGE OF THE  SYSTEM
C ************************************************
C USE  THE IMSLSYS PACKAGE FOR DOUBLE INTEGRATION
C ************************************************
C ASSUMPTIONS :
C
C F IN GHZ AND TAU IN DECIMAL OF NS
C F = 1/T
C COS(X) = 1.0 - X**2/2!
C
C THE 4-QAM CASE
C
      INTEGER  IER
      REAL DBLIN,F,AX,AY,BX,BY,AERR,ERROR,C
      REAL X,Y,Z,AA,DELTA,A1,A2,Y0,Y1,YY,AC,PI
      EXTERNAL F,AY,BY
      PI = 22.0/7.0
      AX = 0.2
      BX = 1.0
      AERR = 0.0001
      C = DBLIN(F,AX,BX,AY,BY,AERR,ERROR,IER)
      WRITE(6,*)C , ERROR
      STOP
      END
      REAL FUNCTION F(X,Y)
      REAL X,Y,Z,AA,DELTA,A1,A2,Y0,Y1,YY,AC,PI
      INTEGER M
      PI=22.0/7.0
      M = 4
C
C
      DELTA = X*(1.0-((2.0*PI*0.1*Y)**2)/2.0)
      AA= SQRT( 1.5*Z/(FLOAT(M)-1.0))
      A1= ((1.0+DELTA)*AA)
      A2= ((1.0+DELTA-2.0*DELTA*0.1*Y/
     *(ALOG10(FLOAT(M))))*AA)
      Y0= (EXP(-A1**2))/(SQRT(4.0*A1))
      Y1= (EXP(-A2**2))/(SQRT(4.0*A2))
      YY = 0.5*(Y0+Y1)
      F = YY*Y*EXP(-Y)
      RETURN
      END
      REAL FUNCTION AY(X)
```

```
      REAL X
      AY= 0.3
      RETURN
      END
      REAL FUNCTION BY(X)
      REAL X
      BY= 0.7
      RETURN
      END
C
C THE 16-QAM CASE
C
C
      INTEGER  IER
      REAL DBLIN,F,AX,AY,BX,BY,AERR,ERROR,C
      REAL X,Y,Z,AA,DELTA,A1,A2,Y0,Y1,YY,AC,PI
      EXTERNAL F,AY,BY
      PI = 22.0/7.0
      AX = 0.2
      BX = 1.0
      AERR = 0.0001
      C = DBLIN(F,AX,BX,AY,BY,AERR,ERROR,IER)
      WRITE(6,*)C , ERROR
      STOP
      END
      REAL FUNCTION F(X,Y)
      REAL X,Y,Z,AA,DELTA,A1,A2,Y0,Y1,YY,AC,PI
      INTEGER M
      PI=22.0/7.0
      M = 16
      Z = 20.0
C
C
C
      DELTA = X*(1.0-((2.0*PI*0.1*Y)**2)/2.0)
      AA= SQRT( 1.5*Z/(FLOAT(M)-1.0))
      AM = 4.0
      A1= (1.+3.*DELTA)*AA
      A2= (1.+3.*DELTA-DELTA*(0.1*Y/AM))*AA
      A3= (1.+3.*DELTA-2.*DELTA*(0.1*Y/AM))*AA
      A4= (1.+3.*DELTA-3.*DELTA*(0.1*Y/AM))*AA
      A10= (1.+DELTA)*AA
      A11= (1.-DELTA)*AA
      A20= (1.+DELTA+DELTA*(0.1*Y/AM))*AA
      A21= (1.-DELTA-DELTA*(0.1*Y/AM))*AA
      A21= (1.-DELTA+DELTA*(0.1*Y/AM))*AA
      A30= (1.+DELTA-DELTA*(0.1*Y/AM))*AA
      A31= (1.-DELTA+DELTA*(0.1*Y/AM))*AA
      A40= (1.+DELTA-2.*DELTA*(0.1*Y/AM))*AA
      A41= (1.-DELTA+2.*DELTA*(0.1*Y/AM))*AA
      Y1= (EXP(-A1**2))/(SQRT(16.0*A1))
```

```
 Y2= (EXP(-A2**2))/(SQRT(16.0*A2))
 Y3= (EXP(-A3**2))/(SQRT(16.0*A3))
 Y4= (EXP(-A4**2))/(SQRT(16.0*A4))
 Y10= (EXP(-A10**2))/(SQRT(16.0*A10))
 Y11= (EXP(-A11**2))/(SQRT(16.0*A11))
 Y20= (EXP(-A20**2))/(SQRT(16.0*A20))
 Y21= (EXP(-A21**2))/(SQRT(16.0*A21))
 Y30= (EXP(-A30**2))/(SQRT(16.0*A30))
 Y31= (EXP(-A31**2))/(SQRT(16.0*A31))
 Y40= (EXP(-A40**2))/(SQRT(16.0*A40))
 Y41= (EXP(-A41**2))/(SQRT(16.0*A41))
 PE1=(1./4.)*(Y1+Y2+Y3+Y4+Y10+Y11+
*Y20+Y21+Y30+Y31+Y40+Y41)/SQRT(PI)
 YY= 2.0*PE1 - PE1**2
 F = YY*Y*EXP(-Y)
 RETURN
 END
 REAL FUNCTION AY(X)
 REAL X
 AY= 0.3
 RETURN
 END
 REAL FUNCTION BY(X)
 REAL X
 BY= 0.7
 RETURN
 END
```

S/N in dB	Outage For 4-QAM
11.76	0.784925192E-07
14.77	0.973126961E-12
16.53	0.142946458E-15
17.78	0.606699498E-20

S/N in dB	Outage For 16-QAM
13.01	0.258216634E-01
14.77	0.208016746E-01
17.78	0.136392638E-01
19.03	0.112372227E-01
20.00	0.960522518E-02